

$$\frac{\|\Delta \mathbf{x}\|}{\|\mathbf{x}\|} \leq \frac{\text{cond}(\mathbf{A})}{1 - \text{cond}(\mathbf{A}) \frac{\|\Delta \mathbf{A}\|}{\|\mathbf{A}\|}} \left( \frac{\|\Delta \mathbf{A}\|}{\|\mathbf{A}\|} + \frac{\|\Delta \mathbf{b}\|}{\|\mathbf{b}\|} \right)$$

## 1 Teoria dos erros

$$u = \beta^{1-n} \quad \text{arredondamento por corte}$$

$$u = \frac{1}{2}\beta^{1-n} \quad \text{arredondamento simétrico}$$

## 2 Equações não-lineares

Método da secante:

$$x_{m+1} = x_m - f(x_m) \frac{x_m - x_{m-1}}{f(x_m) - f(x_{m-1})}$$

$$e_{m+1} = -\frac{f''(\xi_m)}{2f'(\eta_m)} e_{m-1} e_m$$

$$\eta_m \in \text{int}(x_{m-1}, x_m) \quad \xi_m \in \text{int}(x_{m-1}, x_m, z)$$

Método de Newton:

$$x_{m+1} = x_m - \frac{f(x_m)}{f'(x_m)}$$

$$e_{m+1} = -\frac{f''(\xi_m)}{2f'(x_m)} e_m^2 \quad \xi_m \in \text{int}(z, x_m)$$

Método do ponto fixo:

$$x_{m+1} = g(x_m)$$

$$|e_{m+1}| \leq L|e_m|$$

$$|e_{m+1}| \leq \frac{L}{1-L} |x_{m+1} - x_m|$$

$$g'(z) \approx \frac{x_{m+1} - x_m}{x_m - x_{m-1}}$$

$$e_{m+1} = \frac{(-1)^{p-1} g^{(p)}(\xi_m)}{p!} e_m^p \quad \xi_m \in \text{int}(z, x_m)$$

$$g^{(r)}(z) = 0 \quad r = 0, \dots, p-1 \quad g^{(p)}(z) \neq 0$$

$$|e_{m+1}| \leq L_p |e_m|^p \quad L_p = \max_{x \in I} |g^{(p)}(x)/p!|$$

## 3 Sistemas de equações

Normas e condicionamento:

$$\|\mathbf{A}\|_\infty = \max_{1 \leq i \leq n} \sum_{j=1}^n |a_{ij}|$$

$$\|\mathbf{A}\|_1 = \max_{1 \leq j \leq n} \sum_{i=1}^n |a_{ij}|$$

$$\|\mathbf{A}\|_2 = (\rho(\mathbf{A}^T \mathbf{A}))^{1/2}$$

$$\text{cond}(\mathbf{A}) = \|\mathbf{A}\| \|\mathbf{A}^{-1}\|$$

Métodos iterativos:

$$\mathbf{Mx}^{(k+1)} = \mathbf{b} - \mathbf{N}\mathbf{x}^{(k)}$$

$$\mathbf{x} - \mathbf{x}^{(k+1)} = \mathbf{C}(\mathbf{x} - \mathbf{x}^{(k)})$$

$$\mathbf{C} = -\mathbf{M}^{-1}\mathbf{N}$$

Método de Jacobi:

$$x_i^{(k+1)} = (b_i - \sum_{j=1, j \neq i}^n a_{ij} x_j^{(k)}) / a_{ii}$$

Método de Gauss-Seidel:

$$x_i^{(k+1)} = (b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(k+1)} - \sum_{j=i+1}^n a_{ij} x_j^{(k)}) / a_{ii}$$

Método de relaxação SOR:

$$z_i^{(k+1)} = (b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(k+1)} - \sum_{j=i+1}^n a_{ij} x_j^{(k)}) / a_{ii}$$

$$x_i^{(k+1)} = \omega z_i^{(k+1)} + (1 - \omega) x_i^{(k)}$$

Método de Newton:

$$\mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} + \Delta \mathbf{x}^{(k)} \quad \mathbf{J}(\mathbf{x}^{(k)}) \Delta \mathbf{x}^{(k)} = -\mathbf{f}(\mathbf{x}^{(k)})$$

## 4 Interpolação Polinomial

Fórmula interpoladora de Lagrange:

$$p_n(x) = \sum_{i=0}^n f_i l_i(x)$$

$$l_i(x) = \prod_{j=0, j \neq i}^n \frac{x - x_j}{x_i - x_j}$$

Fórmula interpoladora de Newton:

$$p_n(x) = f(x_0) + \sum_{i=1}^n f[x_0, \dots, x_i] (x - x_0) \cdots (x - x_{i-1})$$

$$e_n(x) = f[x_0, \dots, x_n, x] \prod_{i=0}^n (x - x_i)$$

$$f[x_0, \dots, x_k] = \frac{f^{(k)}(\xi)}{k!} \quad \xi \in \text{int}(x_0, \dots, x_k)$$

## 5 Mínimos Quadrados

Equações normais:

$$\begin{bmatrix} (\phi_0, \phi_0) & \cdot & (\phi_0, \phi_m) \\ \cdot & \cdot & \cdot \\ (\phi_m, \phi_0) & \cdot & (\phi_m, \phi_m) \end{bmatrix} \begin{bmatrix} a_0 \\ \cdot \\ a_m \end{bmatrix} = \begin{bmatrix} (\phi_0, f) \\ \cdot \\ (\phi_m, f) \end{bmatrix}$$

## 6 Integração Numérica

$$I_n(f) = \sum_{i=0}^n A_i f(x_i)$$

$$A_i = \int_a^b l_i(x) dx$$

Regra dos trapézios:

$$I_T(f) = \left(\frac{h}{2}\right)[f(a) + f(b)]$$

$$E_T(f) = -\frac{h^3}{12} f''(\xi) \quad \xi \in (a, b)$$

$$I_T^n(f) = h \left[ (f_0 + f_n)/2 + \sum_{i=1}^{n-1} f_i \right]$$

$$\begin{aligned} E_T^n(f) &= -\frac{h^3}{12} \sum_{i=1}^n f''(\xi_i) \\ &= -\frac{h^2(b-a)}{12} f''(\xi) \quad \xi \in (a, b) \end{aligned}$$

Regra de Simpson:

$$I_S(f) = \frac{h}{3} [f(a) + 4f(a+h) + f(b)]$$

$$E_S(f) = -\frac{h^5}{90} f^{(4)}(\xi) \quad \xi \in (a, b)$$

$$I_S^n(f) = \frac{h}{3} \left[ (f_0 + f_n) + 4 \sum_{i=1}^{n/2} f_{2i-1} + 2 \sum_{i=1}^{n/2-1} f_{2i} \right]$$

$$\begin{aligned} E_S^n(f) &= -\frac{h^5}{90} \sum_{i=1}^{n/2} f^{(4)}(\xi_i) \\ &= -\frac{h^4(b-a)}{180} f^{(4)}(\xi) \quad \xi \in (a, b) \end{aligned}$$

Regra do ponto médio:

$$I_M = hf \left( \frac{a+b}{2} \right)$$

$$E_M = \frac{h^3}{24} f''(\xi) \quad \xi \in (a, b)$$

$$I_M^n(f) = h \sum_{i=1}^n f(a + (i-1/2)h)$$

$$E_M^n(f) = \frac{h^2(b-a)}{24} f''(\xi) \quad \xi \in (a, b)$$

## 7 Equações Diferenciais

$$y' = f(x, y) \quad y(x_0) = y_0$$

Método de Euler:

$$\begin{aligned} y_{n+1} &= y_n + h f(x_n, y_n) \\ T_n &= \frac{h^2}{2} y''(\xi_n) \quad \xi \in (x_n, x_{n+1}) \end{aligned}$$

$$|y(x_n) - y_n| \leq \frac{hY_2}{2} \frac{e^{(x_n-x_0)K} - 1}{K}$$

$$K = \max_{x \in [x_0, b]} |f'_y(x, y)|$$

$$Y_2 = \max_{x \in [x_0, b]} |y''(x)|$$

Método baseado na série de Taylor:

$$y_{n+1} = y_n + hf_n + \frac{h^2}{2} [f'_x(x_n, y_n) + f'_y(x_n, y_n) f_n]$$

Métodos de Runge-Kutta:

$$\begin{aligned} y_{n+1} &= y_n + h \left( 1 - \frac{1}{2a_2} \right) f_n + \\ &\quad + \frac{h}{2a_2} f(x_n + a_2 h, y_n + a_2 h f_n) \end{aligned}$$