# Computational approaches to singular free boundary problems in ordinary differential equations

Pedro Lima
*Department of Mathematics,*
*Instituto Superior Tecnico, Universidade de Lisboa*
*Av. Rovisco Pais, 1049-001 Lisboa, Portugal*

plima@math.ist.utl.pt

Many mathematical models in Physics and Mechanics lead to the following free boundary problem: find a real $M > 0$ and a positive solution of the equation

$$(|y'|^{m-2}\, y')' + \frac{N-1}{x}\, |y'|^{m-2}\, y' + a y^q - b y^p = 0, \quad 0 < x < M, \tag{1}$$

which belongs to $C^2((0,M)) \cup C^1([0,M])$ and satisfies the boundary conditions

$$y'(0) = 0, \quad y(M) = y'(M) = 0, \quad M > 0. \tag{2}$$

Concerning the parameters in (1), $N$ is the space dimension ($N \geq 2$), $m > 1$, $p < q$ and $a, b > 0$. The differential operator on the left-hand side of ( 1) is often called the degenerate m-Laplacian.

In [1] a numerical method has been proposed to approximate the solution of this problem, where smoothing variable transformations are applied to deal with the singularities at $x = 0$ and $x = M$. Then the problem is discretized by means of a finite difference scheme.

In the present paper we consider a new numerical approach. First, we rewrite equations (1),(2) in the new variable $z = x/M$, and transform them into an eigenvalue problem, where one of the unknowns is the eigenvalue $\lambda = M^m$. By formulating the original problem in the new variable and applying to the resulting equations the smoothing variable transformations, prescribed in [1], we obtain a new BVP, to which the `bvpsuite` codes [2] can be applied.

The present talk is a joint work with L. Morgado (UTAD, Portugal), E. Weinmüller, S. Schirnhofer, M. Schönbinger (TU Vienna, Austria).

## References

[1] P.M. Lima and M. L. Morgado, *Efficient computational methods for singular free boundary problems using smoothing variable substitutions*, J. Comp. Appl. Math. **236** (2012), 2981-2989.

[2] G. Kitzhofer, O. Koch, G. Pulverer, Ch. Simon, and E. Weinmüller, *The new Matlab code* `bvpsuite` *for the solution of singular implicit BVPs*, J. Numer. Anal. Indust. Appl. Math., 5, 113-134 (2010).