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A STOCHASTIC APPROACH TO NEURAL FIELD EQUATIONS ON UNBOUNDED DOMAINS

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Neural Field Equations (NFE) are a powerful tool for analysing the dynamical behaviour of populations of neurons. The analysis of such dynamical mechanisms is crucially important for understanding a wide range of neurobiological phenomena. As in other sciences, in Neurobiology it is well-known that better consistency with some phenomena can be provided if the effects of random processes in the system are taken into account. In the recent work of Kuhn and Riedler [1], the authors study the effect of additive noise in Neural Field Equations. With this purpose they introduce the stochastic integrodifferential equation

$$dU_t(x) = \left(I(x,t) - \frac{1}{c}U_t(x) + \int_\Omega K(|x-y|)S(U_t(y))dy \right) dt + \epsilon dW_t(x), \tag{1}$$

where $t \in [0,T]$, $x \in \Omega \subset \mathbb{R}^n$, W_t is a Q-Wiener process.

The main goal of the present work is to analyse the effect of noise in certain neural fields with delay, which are known to have Hopf bifurcations. In this case we consider the following modification of equation (1)

$$dU_t(x) = \left(I(x,t) - \frac{1}{c}U_t(x) + \int_\Omega K(|x-y|)S(U_{t-\tau}(y))dy \right) dt + \epsilon dW_t(x), \tag{2}$$

where, as in the deterministic case, τ is a delay, depending on the distance $|x-y|$. Equation (2) is completed with an initial condition of the form

$$U_t(x) = U_0(x,t), \qquad t \in [-\tau_{max}, 0], \quad x \in \Omega, \tag{3}$$

where $U_0(x,t)$ is some given stochastic process, τ_{max} is the maximum value of the delay. In our case, $\Omega = \mathbb{R}$.

In order to approximate equation (2) we apply a numerical scheme which uses the Galerkin method for the space discretization. In this way we obtain a system of stochastic delay differential equations, which are then discretized by the Euler-Maruyama method.

We use this computational algorithm to analyse noise induced changes in the dynamcal behaviour of some neural fields. Some numerical examples are presented and the results are discussed.

REFERENCES

[1] C. Kühn and M.G. Riedler. Large deviations for nonlocal stochastic neural fields. *J. Math. Neurosci.*, **4:1** 2014.1–33.