Numerical Solution of the Two-Dimensional Neural Field Equation

Pedro M Lima[1,2], Evelyn Buckwar[1],
[1] Institute of Stochastics, Johannes Kepler University, 4040- Linz, Austria
[2] CEMAT/Instituto Superior Técnico, University of Lisbon, 1049-Lisboa, Portugal
E-mail: plima@math.ist.utl.pt

We present and discuss a new numerical algorithm for solving the two-dimensional Neural Field Equation with space-dependent delays:

$$c\frac{\partial}{\partial t}V(\bar{x},t) = I(\bar{x},t) - V(\bar{x},t) + \int_{\Omega} K(|\bar{x}-\bar{y}|)S(V(y,t-\tau(\bar{x},\bar{y}))d\bar{y},$$
$$t \in [0,T], \quad \bar{x} \in \Omega \subset \mathbb{R}^2,$$

used in Neuroscience to describe the evolution of a population of neurons and the interactions between them. A similar equation (without delays) was first introduced by Wilson and Cowan [1], and then by Amari [2]. Here $V(\mathbf{x},t)$ represents the post-synaptic membrane potential at instant t and position $\mathbf{x}$. The function I represents external sources of excitation and S describes the dependence between the firing rate of the neurons and their membrane potential (typically it is a function of sigmoidal type). The kernel function $K(|\mathbf{x}-\mathbf{y}|)$ describes the connectivity between the neurons at positions x and y. The delay $\tau(x,y)$ takes into the consideration the time spent by an electrical signal to travel between these two positions.

The new numerical method presented in this work is based on an implicit second order method for discretisation in time and uses Gaussian quadratures for space integration. We use low-rank methods to reduce the computational effort, which enables to reduce very significantly the dimensions of the involved matrices, without affecting the final accuracy of the method.

This algorithm is targeted directly to the application in Neuroscience and Robotics.

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References

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