

GEOMETRY OF $6j$ SYMBOLS

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1. KÄHLER QUANTIZATION AND THE PONZANO-REGGE-ROBERTS FORMULA.

I will review Kähler quantization, introduce the Racah-Wigner $6j$ symbols, and derive the Ponzano-Regge-Roberts formula for their semiclassical asymptotics from functorial properties of Bohr-Sommerfeld sequences. This is a partial answer to the first question at the end of Roberts paper, is it possible to give a two-dimensional proof of the formula?

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2. SEMICLASSICAL ASYMPTOTICS OF QUANTUM $6j$ SYMBOLS.

I will discuss joint work with Yuka Taylor on the asymptotics of $6j$ symbols for $U_q(\mathfrak{sl}_2)$ for $|q| = 1$ (spherical case) or q real (hyperbolic case). There are eight different cases; we solved seven of them. The last, classically forbidden case, is expected to show exponential decay, only if q is a primitive root of unity.

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3. PUZZLES ON THE ASYMPTOTICS OF TURAEV-VIRO.

Witten and others have predicted that the asymptotics of the Reshetikhin-Turaev and therefore also the Turaev-Viro invariant is a sum over flat connections, involving the torsion, spectral flow, and Chern-Simons invariant.

I will discuss what results from substituting the asymptotic formula for $6j$ symbols into the Turaev-Viro invariant, and applying stationary phase.

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