Integrais e Cálculo Vectorial

1. Integrais

1. **Múltiplos:**

$$\int \cdots \int_V f(x)\, dx_1 \dots dx_n = \int \cdots \int_T f(\mathbf{g}(t))\, |\det D\mathbf{g}|\, dt_1\, \dots\, dt_n$$

2. **De linha:**
   (i) Campos escalares:

$$\int_C f = \int_a^b f(\mathbf{g}(t))\, \|\mathbf{g}'(t)\|\, dt$$

   (ii) Campos vectoriais:

$$\int_C \mathbf{F} \cdot d\mathbf{g} = \int_a^b \mathbf{F}(\mathbf{g}(t)) \cdot \mathbf{g}'(t)\, dt$$

3. **De superfície:**
   (i) Campos escalares:

$$\begin{aligned}\int_S f &= \int_T f(\mathbf{g}(t))\sqrt{\det(D\mathbf{g}^t D\mathbf{g})}\, dt \\ &= \int_T f(\mathbf{g}(t))\, \|(D_1\mathbf{g}(t) \times D_2\mathbf{g}(t))\|\, dt\end{aligned}$$

   (ii) Campos vectoriais:

$$\int_S \mathbf{F} \cdot \mathbf{n} = \int_T \mathbf{F}(\mathbf{g}(t)) \cdot (D_1\mathbf{g}(t) \times D_2\mathbf{g}(t))\, dt$$

2. Teoremas

1. **Teorema Fundamental do Cálculo para Integrais de Linha:**

$$\int_C \nabla\Phi \cdot d\mathbf{g} = \Phi(\mathbf{g}(b)) - \Phi(\mathbf{g}(a))$$

2. **Teorema de Green:**

$$\oint_{\partial D} \mathbf{F} \cdot d\mathbf{g} \equiv \oint_{\partial D} P dx + Q dy = \int_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\, dy$$

3. **Teorema da Divergência:**

$$\int_V \operatorname{div} \mathbf{F} = \oint_{\partial V} \mathbf{F} \cdot \mathbf{n}$$

4. **Teorema de Stokes:**

$$\int_S \operatorname{rot} \mathbf{F} \cdot \mathbf{n} = \oint_{\partial S} \mathbf{F} \cdot \mathbf{dg}$$

5. $\operatorname{rot} \nabla\phi = \mathbf{0}$
6. $\operatorname{div} \operatorname{rot} \mathbf{F} = 0$
7. $\operatorname{rot} \mathbf{F} = \mathbf{0}$ + domínio simplesmente conexo $\Rightarrow \mathbf{F} = \nabla\phi$
8. $\operatorname{div} \mathbf{F} = 0$ + domínio em estrela $\Rightarrow \mathbf{F} = \operatorname{rot} \mathbf{A}$