STRING THEORY &

NONPERTURBATIVE GAUGE THEORY

N=1 THEORIES:

$$\begin{cases} Q_\alpha = \dfrac{\partial}{\partial\theta_\alpha} - i\sigma^\mu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_\mu \\ D_\alpha = \dfrac{\partial}{\partial\theta_\alpha} + i\sigma^\mu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_\mu \end{cases}$$

$$\begin{cases} \bar Q_{\dot\alpha} = -\dfrac{\partial}{\partial\bar\theta_{\dot\alpha}} + i\theta^\alpha\sigma^\mu_{\alpha\dot\alpha}\partial_\mu \\ \bar D_{\dot\alpha} = -\dfrac{\partial}{\partial\bar\theta_{\dot\alpha}} - i\theta^\alpha\sigma^\mu_{\alpha\dot\alpha}\partial_\mu \end{cases}$$

EASIER NOTATION:

$$i\sigma^\mu_{\alpha\dot\alpha}\partial_\mu \Longrightarrow \partial_{\alpha\dot\alpha}$$

CHIRAL SUPERFIELD: $\bar D_{\dot\alpha}\Phi = 0 \Rightarrow \bar Q_{\dot\alpha}\Phi = 2\theta^\alpha\partial_{\alpha\dot\alpha}\Phi$

$$\Phi \simeq \Phi(\underbrace{x_{\alpha\dot\alpha} + i\theta_\alpha\bar\theta_{\dot\alpha}}_{y_{\alpha\dot\alpha}}, \theta_\alpha) = \varphi(y) + \theta^\alpha\psi_\alpha(y) + \theta^\alpha\theta_\alpha F$$

F−TERM

VECTOR SUPERFIELD V:

$$\begin{cases} W_\alpha = i\bar{D}^{\dot\alpha}\bar{D}_{\dot\alpha}\left[e^{-V}D_\alpha e^{V}\right] = \lambda_\alpha + \theta^\beta F_{\alpha\beta} + \dots \\ W_{\dot\alpha} = e^{-V}\bar{W}_{\dot\alpha}e^{V} = e^{-V}(-W_\alpha)^\dagger e^{V} \\ \nabla^\alpha W_\alpha + \nabla^{\dot\alpha}W_{\dot\alpha} = 0 \end{cases}$$

GAUGE TRANSFORMATIONS:

$$\begin{cases} \Phi' = e^{i\Lambda}\Phi \\ \bar\Phi' = \bar\Phi e^{-i\bar\Lambda} \\ e^{V'} = e^{i\bar\Lambda}e^{V}e^{-i\Lambda} \end{cases} \Rightarrow \bar\Phi e^{V}\Phi \quad \text{GAUGE INVARIANT}$$

ACTION:

$$S = \int d^4\theta\, d^4x\, \underbrace{K(\Phi^i,\bar\Phi^{\bar j})}_{\text{kähler potential}} + \int d^4x\, d^2\theta\, \underbrace{W(\Phi^i)}_{\text{superpotential}} + \text{c.c.}$$

$$\Downarrow$$

$$V_{SCALAR} \sim \partial_i W\left[\partial_i\partial_{\bar j}K\right]^{-1}\partial_{\bar j}\bar{W} \qquad \text{FROM } F$$

DETERMINES MASSES

ACTION:

$$S = \int d^4x\, d^4\theta\, \mathrm{Tr}\left[e^{-V}\bar{\Phi}e^{V}\Phi\right]$$

$$+ \frac{1}{g^2_{YM}}\int d^4x\, d^2\theta\, \mathrm{Tr}\left[W^\alpha W_\alpha\right] + \text{c.c.}$$

$$+ \int d^4x\, d^2\theta\, \mathrm{Tr}(W[\Phi]) + \text{c.c.}$$

CONSIDER CORRELATION FUNCTIONS OF CHIRAL OPERATORS

$$\langle \mathcal{O}_1 \mathcal{O}_2 \ldots \mathcal{O}_k \rangle$$

① $\{\bar{Q}_{\dot\alpha}, \varphi\} = 0$

$\{\bar{Q}_{\dot\alpha}, \psi_\alpha\} = 2\partial_{\alpha\dot\alpha}\varphi \quad \Longrightarrow$

$$\langle (\partial_{\alpha\dot\alpha}\mathcal{O}_1)\mathcal{O}_2 \ldots \mathcal{O}_k \rangle = \frac{1}{2}\langle \{\bar{Q}_{\dot\alpha}, \psi'_\alpha\}\mathcal{O}_2 \ldots \mathcal{O}_k\rangle$$

$$= 0$$

CORRELATION FUNCTIONS ARE INDEPENDENT OF POSITION

② $$[\bar{Q}^{\dot\alpha}, [\nabla_{\alpha\dot\alpha}, \mathcal{O}\}\} = [[\bar{Q}^{\dot\alpha}, \nabla_{\alpha\dot\alpha}], \mathcal{O}\} + \mathcal{O}(\theta)$$
$$= [[\bar\nabla^{\dot\alpha}, \nabla_{\alpha\dot\alpha}], \mathcal{O}\} + \mathcal{O}(\theta)$$
$$= \underbrace{[W_\alpha, \mathcal{O}\}}_{\text{COMMUTATOR IN ADJOINT}} + \mathcal{O}(\theta)$$

$\Rightarrow [W_\alpha, \mathcal{O}\}$ DECOUPLES IN CORRELATORS

③ $$\bar{m}\frac{\partial}{\partial \bar{m}} \langle \mathcal{O}_1 \ldots \mathcal{O}_k \rangle =$$
$$\bar{m}\int \langle \mathcal{O}_1 \ldots \mathcal{O}_k \, \bar{X}_{TOP}(x) \rangle d^4x$$
$$\bar{m}\int \langle \mathcal{O}_1 \ldots \mathcal{O}_k \{\bar{Q}^{\dot\alpha}, \bar\Psi_{\dot\alpha}\} \rangle d^4x = 0$$

NO DEPENDENCE ON ANTIHOLOMORPHIC PARAMETERS

④ $$\partial_\Phi W = \bar{D}_{\dot\alpha} \bar{D}^{\dot\alpha} \bar\Phi$$

$\Rightarrow \partial_\Phi W$ DECOUPLES IN CORRELATORS

* THIS DEFINES THE CHIRAL RING

COMMENTS:

- ALL THIS ASSUMES UNBROKEN SUSY
- $\langle \mathcal{O}_1 \cdots \mathcal{O}_k \rangle = \langle \mathcal{O}_1 \rangle \langle \mathcal{O}_2 \rangle \cdots \langle \mathcal{O}_k \rangle$

  USING CLUSTERING (UNIQUE GROUND STATE)

- PERTURBATION THEORY GENERATES NO CORRECTIONS TO SUPERPOTENTIAL (NON-RENORMALIZATION THEOREM)

  UNLESS $S_{eff} \to \int d^4\theta\, \phi^2 \frac{D^2}{\Box} \phi$ OR SO; IR?

- POWERFUL TOOL: $U(1)$ — SYMMETRIES

  HYPER: $\begin{cases} \varphi \to \varphi \cdot e^{i\beta} \\ \psi^\alpha \to \psi^\alpha \end{cases}$ VECTOR $\begin{cases} \lambda^\alpha \to e^{i\beta}\lambda^\alpha & \text{GLUINO} \\ A_\mu \to A_\mu \end{cases}$

  $$U(1)_R \xrightarrow{\text{INSTANTONS}} \mathbb{Z}_{2N} \xrightarrow{\text{DYNAMICAL}} \mathbb{Z}_2$$

  $U(1)_R$ CAN BE RESTORED BY GIVING

  $\Lambda \sim e^{-8\pi^2/g^2(\mu)}$ APPROPRIATE TRANSFORMATION RULE

- ALSO: CLASSICAL LIMITS, HOLOMORPHY ARE POWERFUL TOOLS

CHIRAL QUANTITIES ARE SIMPLE

CONSIDER IN GENERAL LOW-ENERGY EFFECTIVE

ACTION $S_{eff}[X_i, g_i, \Lambda]$

LIGHT FIELDS

COUPLINGS

1PI VERSUS WILSONIAN ?

↳ PROBLEMATIC WITH INTERACTING MASSLESS PARTICLES : USE WILSONIAN.

FOCUS ON $W_{eff} \in S_{eff}$: PROVIDES ALL CHIRAL INFORMATION BY EXTREMIZING AND CLUSTERING ETC.

INTEGRATING IN

INTRILIGATOR & SEIBERG

SUPPOSE $\{X_i\}$ IS THE SET OF LIGHT CHIRAL FIELDS.

TAKE $W = W_0 + \sum g_i X_i$ AS CLASSICAL SUPERPOTENTIAL.

- NEXT, COMPUTE

$$W_q[g_i, \Lambda]$$

VIEWING g_i AS BACKGROUND CHIRAL SUPERFIELDS

- NEXT, COMPUTE

$$W_q[x^i] = \left(W_q(g_i) - \sum g_i X_i\right)\Big|_{g_i}$$

LEGENDRE TRANSFORM EXTREMUM

- THEN DEFINE

$$\boxed{W_{eff} = W_q(X_i) + \sum g_i X_i}$$

HAS 1) RIGHT EXTREMA

2) COMPATIBLE WITH "LINEARITY PRINCIPLE" ($\leftrightarrow$ NONRENORMALIZATION THEOREM)

THIS IS THE RIGHT ANSWER IF

1) THEORY WITH SOURCES HAS A MASS GAP: AVOIDS IR-PROBLEM

2) SUSY IS UNBROKEN

$$\left[\text{cf: } W = \lambda \varphi_0 + m \varphi_1 \varphi_2 + g \varphi_0 \varphi_1 \varphi_2 \quad \text{FAYET-O'Raifeartaigh model} \right]$$

3) ALL LIGHT FIELDS SHOULD BE INCLUDED

4) OK TO FIND VEVS OF MASSIVE (COMPOSITE) FIELDS, BUT SHOULD NOT THINK OF THEM AS DESCRIBING MASSIVE PARTICLES

APPLICATION

$$S = \mathrm{Tr}\left[W^\alpha W_\alpha\right] = \mathrm{Tr}\left[\lambda^\alpha \lambda_\alpha\right] + \ldots$$

GLUINO BILINEAR SUPERFIELD

WHAT IS $W_{EFF}[S]$?? FOR PURE $N=1$ THEORIES ?

IN THE ACTION, TREE LEVEL COUPLING IS

$$W = \underbrace{3N}_{\hookrightarrow \text{ONE-LOOP } \beta\text{-FUNCTION}} . \log\Lambda . S \qquad \left(= \frac{1}{g_{YM}^2} S\right)$$

HONEST INSTANTON CALCULATION

$$\langle S^N \rangle = \Lambda^{3N} \implies \langle S \rangle = \Lambda^3 \qquad (*)$$

SUPPOSE AT MINIMUM WE GET $W_q(\Lambda)$

THEN FULL RESULT

$$W_{eff} = \left[W_q(u) - 3N . \log u . S \right] \Big|_{u_{extr}} + 3N . \log\Lambda . S$$

COMPUTE MINIMA OF W_{EFF}:

$$\begin{cases} W_q'(u) - 3N \frac{S}{u} = 0 \\ -3N \log u + 3N \log\Lambda = 0 \end{cases} \Rightarrow \begin{cases} S = \frac{1}{3N} u W_q'(u) \Big|_{u=\Lambda} \\ u = \Lambda \end{cases}$$

THIS IS COMPATIBLE WITH $(*)$ IF

$$\Lambda^3 = \frac{1}{3N} \Lambda W_q'(\Lambda) \Rightarrow W_q(\Lambda) = N\Lambda^3$$

THIS THEN YIELDS

$$W_{eff} = S\left[\log\frac{\Lambda^{3N}}{S^N} + N\right]$$

WHICH IS THE CELEBRATED VENEZIANO-YANKIELOWICZ ANSWER

THIS IS A GOOD EFFECTIVE ACTION IF S IS FUNDAMENTAL, OTHERWISE JUST A TOOL TO SEE WHETHER $\langle S\rangle =$?

[illegible]

EXPLAINED

$$W_{VY}[S] = S\left[\operatorname{LOG}\left(\frac{\Lambda^{3N}}{S^N}\right) + N\right]$$

- ALTERNATIVE DERIVATION OF W_{MIN}:

COMPACTIFY ON S^1

$$\left.\begin{matrix} A_0 \\ A_1 \\ A_2 \end{matrix}\right\} \text{3d gauge field} \xrightarrow{\text{dualize}} \text{scalar}$$

$$A_3 \longrightarrow \text{scalar}$$

TOGETHER: COMPLEX SCALAR

GENERICALLY $U(N) \to U(1)^N$ SCALARS Y_i

OBTAIN [FROM MONOPOLES]

$$W = e^{Y_1 - Y_2} + e^{Y_2 - Y_3} + \dots + \Lambda^3 e^{Y_N - Y_1}$$

EXTREMUM: $W = N\Lambda^3$