INTERESTING CLASS OF THEORIES:

$N=2$ BROKEN TO $N=1$

$N=2$ CONSISTS OF AN $N=1$ VECTOR MULTIPLET AND OF AN $N=1$ ADJOINT SCALAR FIELD Φ

BREAK $N=2 \to N=1$ BY ADDING

$$W_{TREE}(\Phi) = \sum \frac{g_k}{k+1} \mathrm{Tr}(\Phi^K)$$

SUPPOSE $W'(\Phi) = \mathrm{Tr}((\Phi - p_1)(\Phi - p_2) \cdots (\Phi - p_n))$

CLASSICALLY, SUSY VACUA ARE

$$\Phi = \begin{pmatrix} \left.\begin{matrix} p_1 & & \\ & \ddots & \\ & & p_1 \end{matrix}\right\} N_1 & & & \\ & \left.\begin{matrix} p_2 & & \\ & \ddots & \\ & & p_2 \end{matrix}\right\} N_2 & & \\ & & \ddots & \\ & & & \left.\begin{matrix} p_n & & \\ & \ddots & \\ & & p_n \end{matrix}\right\} N_n \end{pmatrix}$$

WHICH BREAKS $U(N) \to \prod U(N_i)$

IDEA: $S_i \sim \mathrm{Tr}_{U(N_i)}(W^\alpha W_\alpha)$ ARE

GOOD LOW-ENERGY FIELDS

CLAIM: $W[S_i] = \sum' S_i \left(\log \left(\frac{\Lambda^{3N_i}}{S_i^{N_i}} \right) + N_i \right) + W_{pert}[S_i]$

PROOF: WEAKER, WE CANNOT INTEGRATE

IN THE S_i : NO SOURCE

$$S_i \sim \oint_{P_i} \mathrm{Tr}\left(\frac{W^\alpha W_\alpha}{(z - \Phi)} \right) dz$$

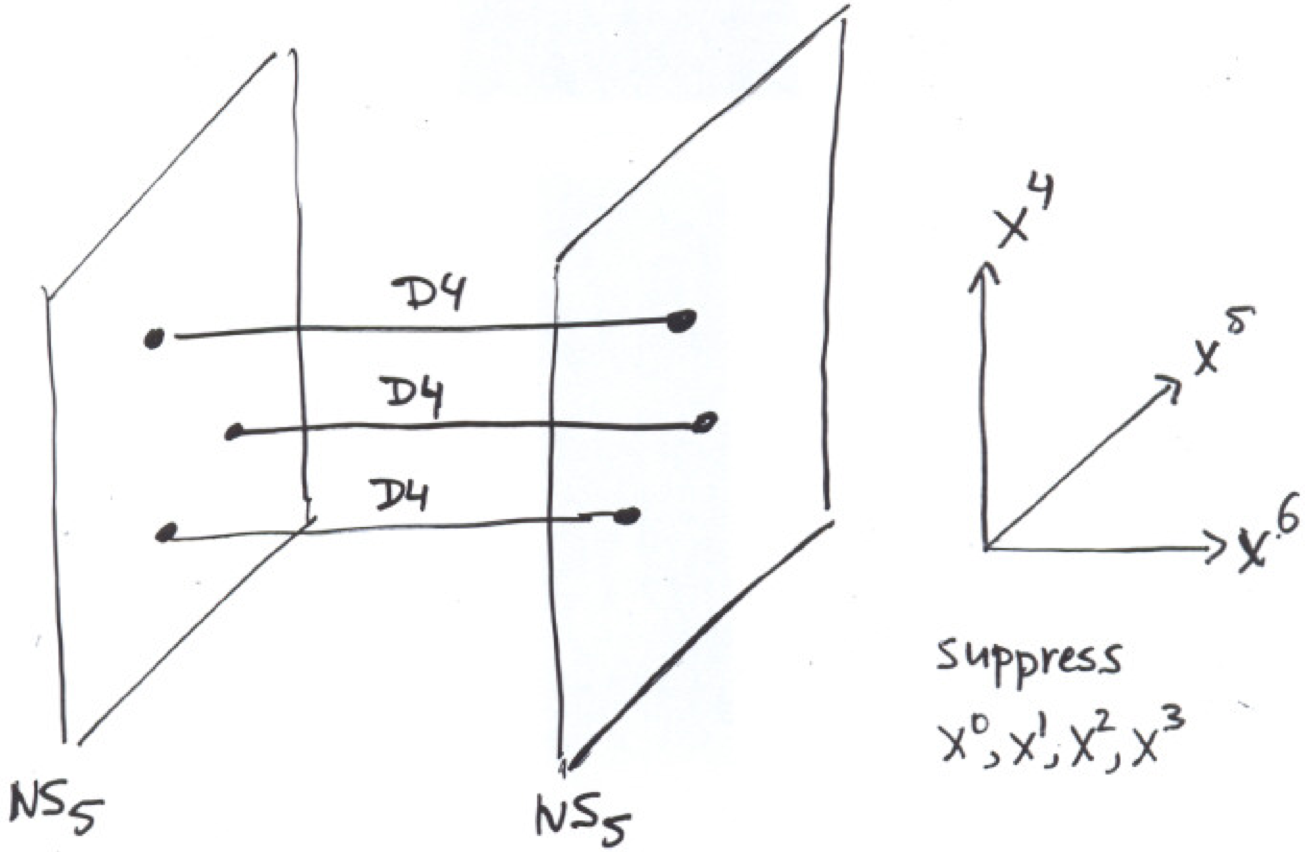
$W[S_i]$ HAS THE RIGHT EXTREMA

$W_{PERT}[S_i]$ CAN BE OBTAINED FROM

- KONISHI ANOMALY
- MATRIX INTEGRALS
- CY GEOMETRY

BRANE CONSTRUCTION OF $N=2$ THEORIES

Witten



THE D4-BRANES SUPPORT A PURE $N=2$ $U(N)$ GAUGE THEORY, $N = \#$ D4 BRANES

$\Phi : \begin{pmatrix} & A_\mu & \\ \lambda^\alpha & & \psi^\alpha \\ & \varphi & \end{pmatrix}$

POINT ON COULOMB BRANCH $\longleftrightarrow$

EIGENVALUES OF Φ : ADJOINT SCALAR SUPERFIELD $\longleftrightarrow$

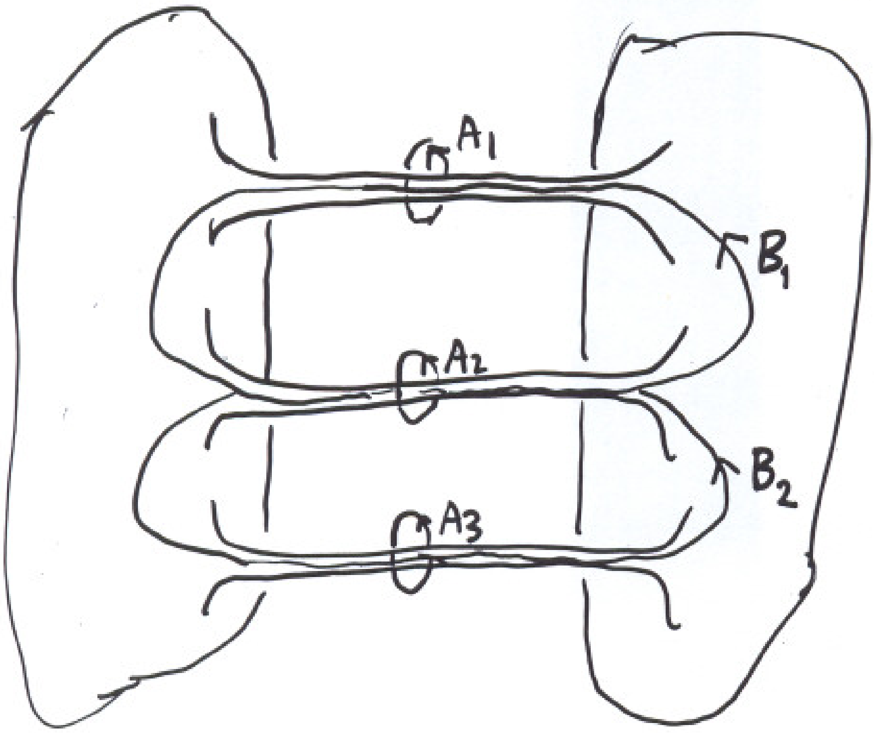
LOCATIONS OF D4-BRANES IN x^4, x^5

LIFT TO M-THEORY:

EXTRA CIRCLE APPEARS, RADIUS R

D4 BRANE $\Longleftrightarrow$ M5 BRANE WRAPPED ON S^1

NS5 BRANE $\Longleftrightarrow$ M5 BRANE NOT WRAPPED ON S^1



SINGLE M5-BRANE WRAPPED ON RIEMANN SURFACE Σ

$$v = x^4 + ix^5$$

$$t = \exp\left(-(x^6 + ix^{10})/R\right)$$

$$\Sigma: \quad t^2 - 2P_N(v)t + \Lambda_{N=2}^{2N} = 0 \equiv \text{Seiberg–Witten curve}$$

$$P_N(v) = \det(v - \Phi)$$

THE (2,0) THEORY ON THE M5 BRANE

REDUCES TO THE LOW-ENERGY EFFECTIVE

ACTION OF QUANTUM N=2 SYM

"MQCD"

AGREEMENT ONLY FOR BPS QUANTITIES

$$\underbrace{B_{\mu i}}_{\text{self-dual 6d two form}} = \sum_{k} \underbrace{\omega_i^{(k)}}_{\text{holomorphic one-forms on } \Sigma} \underbrace{A_\mu^{(k)}}_{\text{4d gauge fields}}$$

PERIOD MATRIX

$$\tau_{kl} = \int_{B_k} \omega^{(l)}$$

YIELD LOW-ENERGY GAUGE COUPLINGS

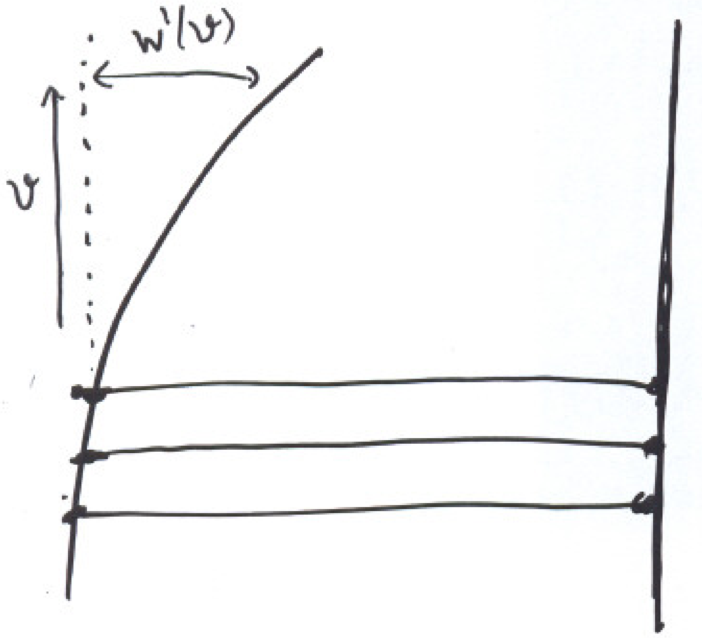
BREAK TO $\mathcal{N}=1$

$$\Delta \mathcal{L} = \mathrm{Tr}\left(\int d^4x\, d^2\theta\, W[\Phi]\right)$$

$W[\Phi]$: NON-TRIVIAL SUPERPOTENTIAL

Dijkgraaf-Vafa

BEND BRANES IN $w = x^8 + i x^9$ DIRECTION



JdB, 02

D4-BRANES DO NOT FIT: OPEN STRING MASS (NS_5-D_4) $\sim W'(v)$

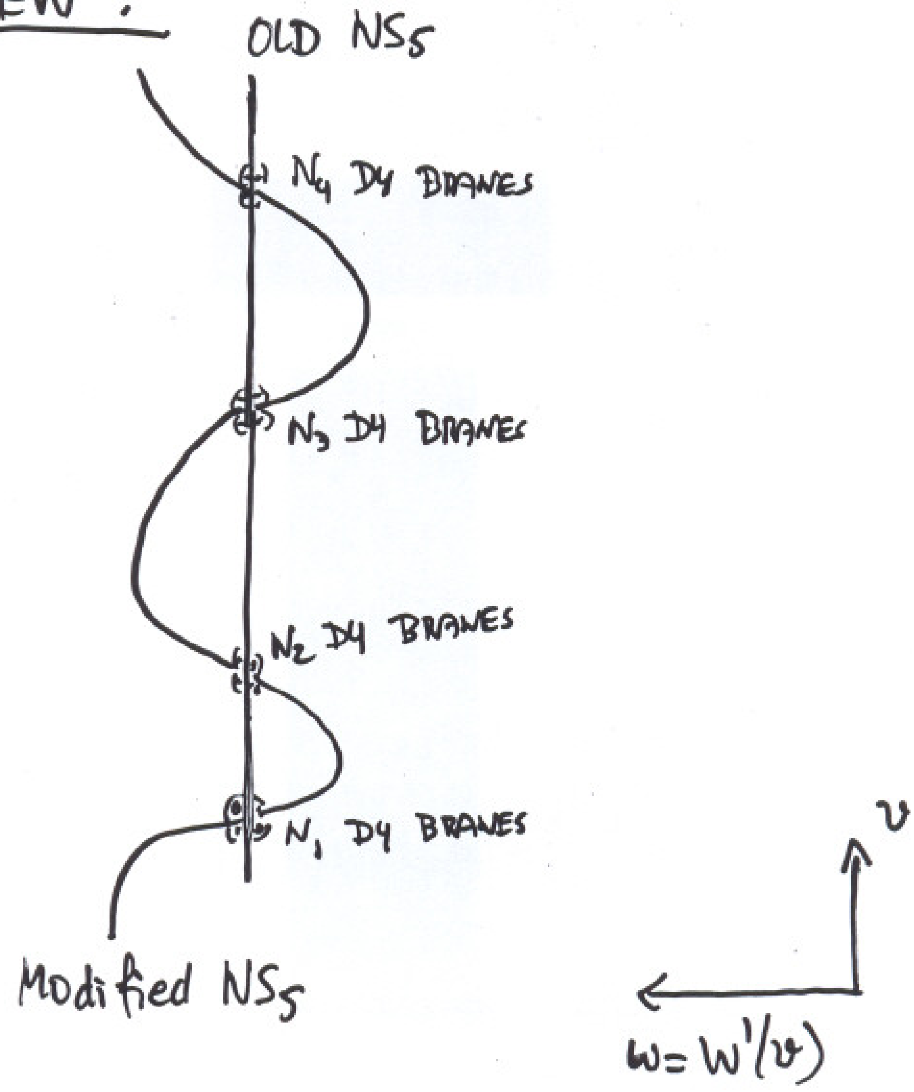
D4-BRANES ONLY FIT WHEN

$$\langle \Phi \rangle = \begin{array}{cl} \begin{pmatrix} p_1 & & & & & & \\ & \ddots & & & & & \\ & & p_1 & & & & \\ & & & p_2 & & & \\ & & & & \ddots & & \\ & & & & & p_2 & \\ & & & & & & \ddots \\ & & & & & & & p_n \\ & & & & & & & & \ddots \\ & & & & & & & & & p_n \end{pmatrix} & \begin{array}{l} \}N_1 \\ \}N_2 \\ \\ \}N_n \end{array} \end{array}$$

WITH p_i THE ROOTS OF $W'(v)=0$

Field theory & branes agree

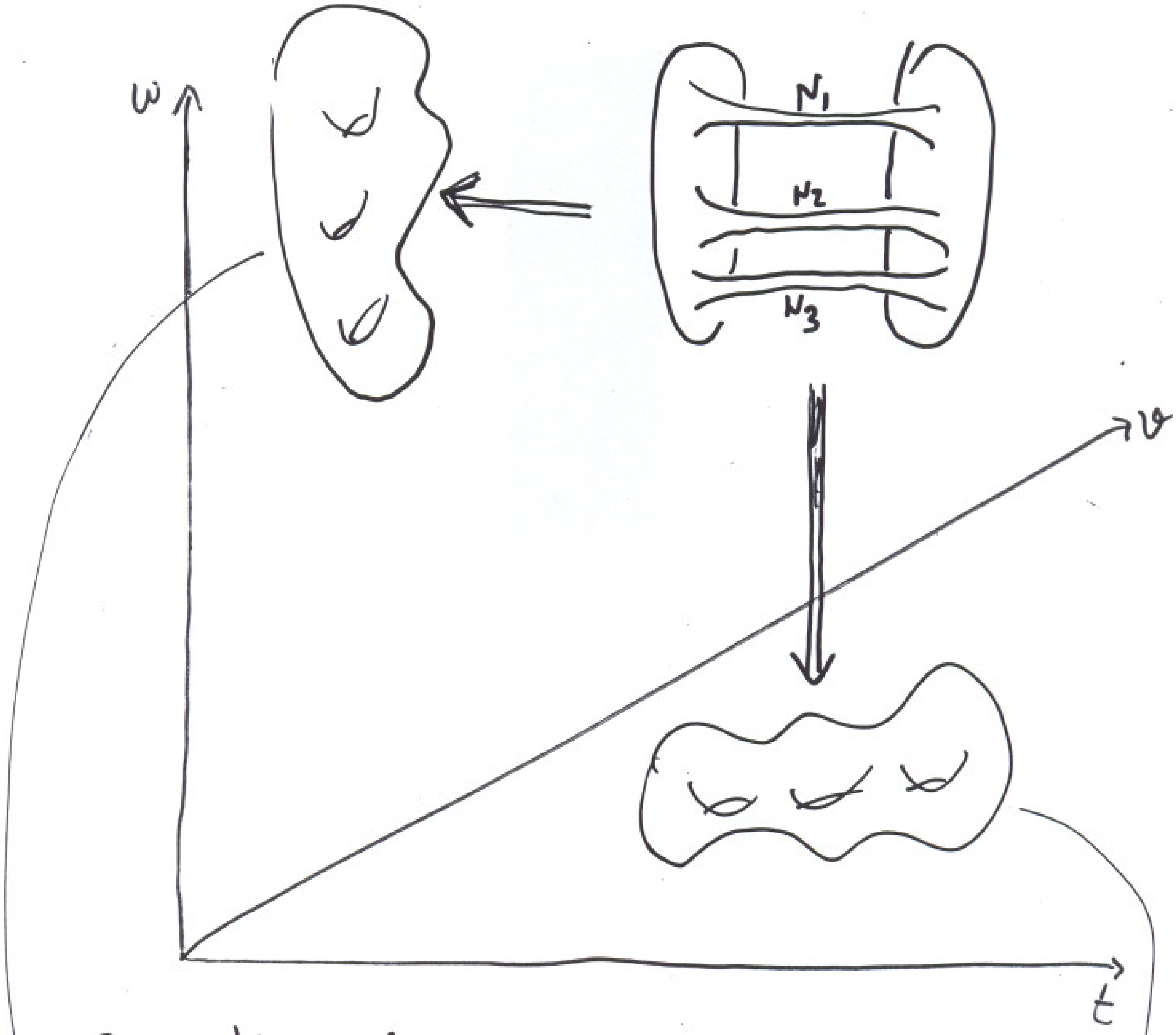
SIDE VIEW:


OLD NS_5
N_4 D4 BRANES
N_3 D4 BRANES
N_2 D4 BRANES
N_1 D4 BRANES
Modified NS_5
v
$w = W'(v)$

THIS BREAKS THE GAUGE GROUP TO

$U(N_1) \times \ldots \times U(N_n)$

LIFT TO M-THEORY : $\Sigma \subset \mathbb{R}^5 \times S^1$



$$w^2 - 2W_m'(v)w - f_{m-1}(v) = 0$$

$$t^2 - 2P_N(v)t + \Lambda_{N=2}^{2N} = 0$$

COMPATIBILITY OF

$$\begin{cases} w^2 - 2W_m'(v)w - f_{m-1}(v) = 0 \\ t^2 - 2P_N(v)t + \Lambda_{N=2}^{2N} = 0 \end{cases}$$

IMPLIES

$$P_N^2(v) - \Lambda_{N=2}^{2N} = S_{N-n}^2(v)\left[G_n^2(v) + f_{n-1}(v)\right]$$

$$W_m'(v)^2 + f_{m-1}(v) = H_{m-n}^2(v)\left[G_n^2(v) + f_{n-1}(v)\right]$$

THESE ALGEBRAIC EQUATIONS DETERMINE COMPLETELY THE LOCATION OF THE ISOLATED QUANTUM VACUA OF THE THEORY

* TWO NATURAL DIFFERENTIALS

Dijkgraaf-Vafa
Cachazo Douglas Seiberg Witten
Cachazo Seiberg Witten

$$\frac{dt}{t} = \mathrm{Tr}_{\text{gauge theory}}\left(\frac{dv}{v-\Phi}\right)$$

$$w\,dv = 2\,\mathrm{Tr}_{\text{matrix theory}}\left(\frac{dv}{v-M}\right) = \frac{-1}{16\pi^2}\mathrm{Tr}_{\text{gauge theory}}\left(\frac{W_\alpha W^\alpha dv}{v-\Phi}\right)$$

$$\int dM\, \mathrm{Tr}\left(\frac{1}{v-M}\right) e^{\frac{-1}{g_s}\mathrm{Tr}(W(M))} \qquad W^\alpha = \lambda^\alpha + \theta_\beta F^{\alpha\beta}$$

BY COMPARING WE OBSERVE THAT IN THE MINIMA

(ALSO NATURAL BY STUDYING DIMENSIONS IN TWISTED (2,0) THEORY)

$$N_i = \frac{1}{2\pi i}\oint_{A_i} \frac{dt}{t} \qquad \tau = \oint_{B_i} \frac{dt}{t}$$

$$S_i = \frac{1}{2\pi i}\oint_{A_i} w\,dv \qquad \frac{\partial \mathcal{F}}{\partial S_i} = \oint_{B_i} w\,dv \qquad (*)$$

HERE, $S_i \sim \mathrm{Tr}_{U(N_i)}(W^\alpha W_\alpha)$ IS THE GAUGINO CONDENSATE SUPERFIELD IN THE (CLASSICALLY) UNBROKEN $U(N_i)$

THERE IS A QUANTUM LOW-ENERGY EFFECTIVE SUPERPOTENTIAL $W[S_i]$, COMPUTED E.G. USING A MATRIX MODEL. MINIMIZING $\left(\frac{\partial W[S_i]}{\partial S_i} = 0\right)$ PUTS S_i EQUAL TO THE VALUES GIVEN IN $(*)$

<u>QUESTION:</u> WHAT IS THE M5-BRANE CONFIGURATION DESCRIBING S_i AWAY FROM SUSY MINIMA, AND HOW DO WE COMPUTE $W[S_i]$ FROM IT ?

JdB, de Haro

NAIVE GUESS:

S_i ARE HOLOMORPHIC (CHIRAL) SUPERFIELDS $\Rightarrow$ LOOK FOR HOLOMORPHIC DEFORMATIONS.

THERE ARE NONE!

CLUE: LOOK AT
$$\begin{cases} N_i = \frac{1}{2\pi i} \oint_{A_i} \frac{dt}{t} & (1) \\ S_i = \frac{1}{2\pi i} \oint_{A_i} w\,dv & (2) \end{cases}$$

IGNORING t, THERE ARE HOLOMORPHIC DEFORMATIONS OF THE CURVE IN (w,v) SPACE AND ENOUGH TO PARAMETRIZE ALL S_i

THERE IS THEN A UNIQUE NONHOLOMORPHIC t SUCH THAT (1) IS STILL SATISFIED

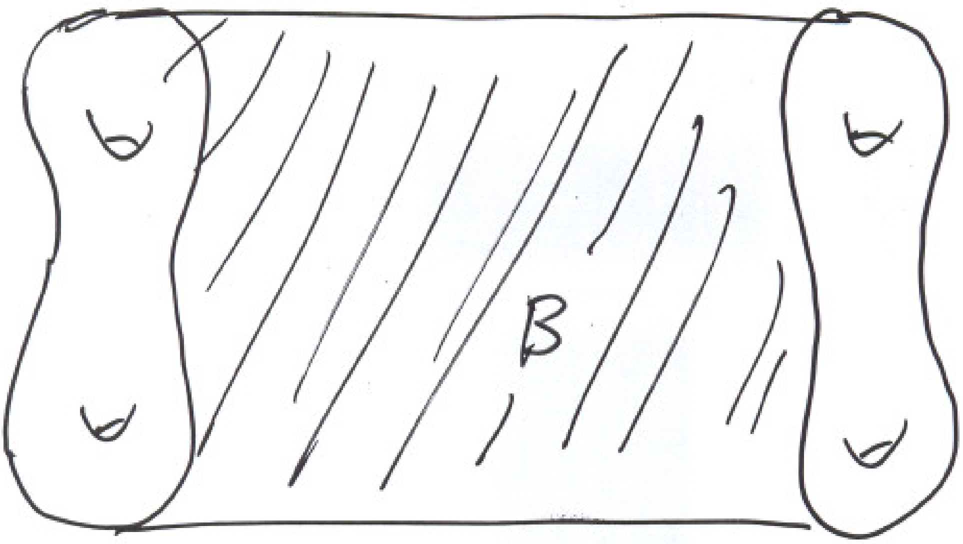
BECAUSE $t = \exp\left[-\frac{x^6 + i x^{10}}{R}\right]$ THE FORM $\frac{dt}{t}$ NEEDS TO HAVE INTEGER PERIODS, WHETHER t IS HOLOMORPHIC OR NOT.

PROPOSAL:

DEFORM Σ HOLOMORPHICALLY IN THE w, v DIRECTION AND THEN DEFINE A (NON-)HOLOMORPHIC t BY THE EQUATIONS $\frac{1}{2\pi i}\oint_{A_i} \frac{dt}{t} = N_i$. (& SIMILAR FOR B_i)

THIS YIELDS A NON-SUPERSYMMETRIC M5-BRANE THAT EXPLICITLY BREAKS SUSY. TO RESTORE SUPERSYMMETRY, WE NEED TO MAKE IT HOLOMORPHIC AND WE RECOVER THE PREVIOUS VACUA

THE SUPERPOTENTIAL



Σ' $\partial B = \Sigma' \# (-\Sigma_0)$ Σ_0 : REFERENCE SURFACE

WITTEN:

$$W(\Sigma') - W(\Sigma_0) = \frac{1}{2\pi i} \int_B \Omega^{(3,0)}$$

$$\Omega^{(3,0)} = R\, dv \wedge dw \wedge \frac{dt}{t}$$

PARTIAL INTEGRATION SUBTLE

$$W = \frac{R}{2\pi i} \int_{\Sigma} \frac{dt}{t} \wedge w\, dv$$

MIRROR TO $\int H \wedge \Omega$

$$= R \sum_{i=1}^{n} N_i \frac{\partial F_0}{\partial S_i} - RNS \log \frac{\Lambda_{N=2}}{\Lambda_0}$$

bilinear identities

cf: Dijkgraaf-Vafa

AGREES!

EXAMPLE (MASS DEFORMED $U(N)$ THEORY)

$$W = \frac{1}{2} m \Phi^2$$

$$(W_1')^2 + f_0 \equiv m^2 v^2 + 2mS$$

THIS GUARANTEES $\frac{1}{2\pi i} \oint_A w\, dv = S$

$$t(v,\bar{v}) = \left[v + \sqrt{v^2 + \frac{2S}{m}} \right]^a \left[\bar{v} + \sqrt{\bar{v}^2 + \frac{2\bar{S}}{m}} \right]^b$$

$$\begin{cases} a - b = N \\ N \log\left(\frac{-2\Lambda_0^2 m}{\Lambda_{N=1}^3} \right) = \log\left[\left(\frac{-2m\Lambda_0^2}{S} \right)^a \times \left(\frac{-2m\bar{\Lambda}_0^2}{\bar{S}} \right)^b \right] \end{cases}$$

$\Rightarrow$ PRECISELY WHEN $S = \Lambda_{N=1}^3$ WE GET $b = 0$.

- CAN NOW EASILY GENERALIZE TO MANY OTHER GAUGE THEORIES FOR WHICH THERE IS A BRANE CONSTRUCTION
- CAN ALSO UNDERSTAND SUPERPOTENTIAL FROM THE DECONSTRUCTION OF THE (2,0) THEORY

SEVERAL FEATURES HAVE A SIMPLE BRANE INTERPRETATION:

- Cachazo Douglas Seiberg Witten POINT OUT THAT EFFECTIVE ACTION CAN BE NICELY WRITTEN IN TERMS OF:

$$V^i = S^i + \eta_\alpha \psi^{i\alpha} + (\eta^\alpha \eta_\alpha) N^i$$

AUXILIARY $N=2$ SUPERFIELD

BUT THIS IS EQUAL TO

$$V^i = \oint_{A_i} dv \Big[\underbrace{\omega + \eta_\alpha \rho^\alpha + (\eta^\alpha \eta_\alpha) \frac{\partial \log t}{\partial v}}_{} \Big]$$

$N=2$ MULTIPLET IN TWISTED (2,0) THEORY

- THE DISTINGUISHED ROLE OF CERTAIN DIFFERENTIAL FORMS IN Cachazo Seiberg Witten NOW HAS A SIMPLE GEOMETRICAL INTERPRETATION

THE KÄHLER POTENTIAL

M5 - BRANE IS $\mathbb{R}^4 \times \Sigma'$

TO GET THE HONEST LOW ENERGY EFFECTIVE ACTION, WE NEED TO LET Σ' VARY OVER $\mathbb{R}^4$. THIS WILL PRODUCE KINETIC TERMS FOR S_i FROM THE FIVEBRANE EFFECTIVE ACTION.

FIND:

$$S = \int d^4x \left(a_{ij}\, \partial_\mu s^i \partial^\mu s^j + b_{ij}\, \partial_\mu s^i \partial^\mu \bar{s}^j + \bar{a}_{ij}\, \partial_\mu \bar{s}^i \partial^\mu \bar{s}^j + \ldots \right)$$

?? NOT COMPATIBLE WITH $N=1$ SUSY ??

$\Rightarrow$ NEED TO MODIFY SPACE-TIME GEOMETRY, NO LONGER $\mathbb{R}^5 \times S^1$, PERHAPS NOT EVEN kähler

$\rightarrow$ Mirror symmetry in the presence of fluxes ($\rightarrow$ Vafa)

$\rightarrow$