Progress

in closed string tachyon condensation on $\mathbb{C}/\mathbb{Z}_n$

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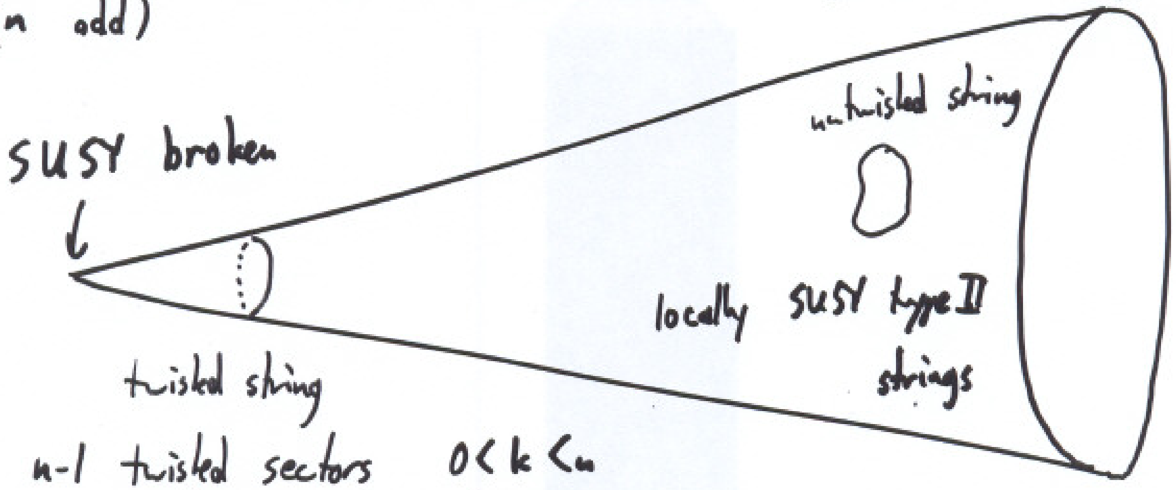
Contents

I. What do we know about the decay of $\mathbb{C}/\mathbb{Z}_n$?

$\mathbb{C}/\mathbb{Z}_n$ orbifold of type II strings: closed string field theory soliton
(n odd)



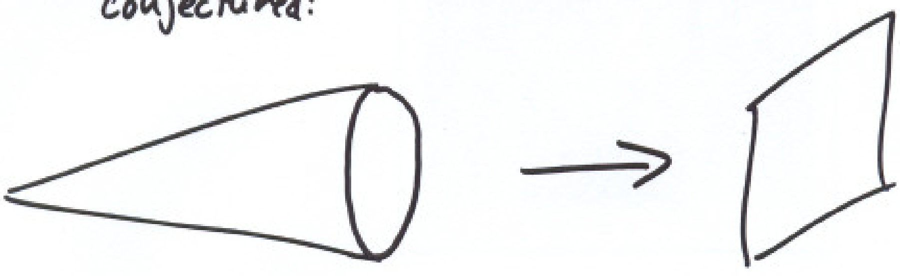
$n-1$ twisted sectors $0<k<n$
each contains ≥ 1 tachyon

$$m_k^2 = -\frac{2}{\alpha'}\begin{cases} 1-\frac{k}{n}, & k \text{ odd} \\ \frac{k}{n}, & k \text{ even} \end{cases}$$

$$E = \frac{2\pi}{\kappa^2}\left(1-\frac{1}{n}\right)$$

non-BPS

Adams, Polchinski, Silverstein (hep-th/0108075)
conjectured:



What is the evidence?

World-sheet RG flow

Vafa (hep-th/0111051)

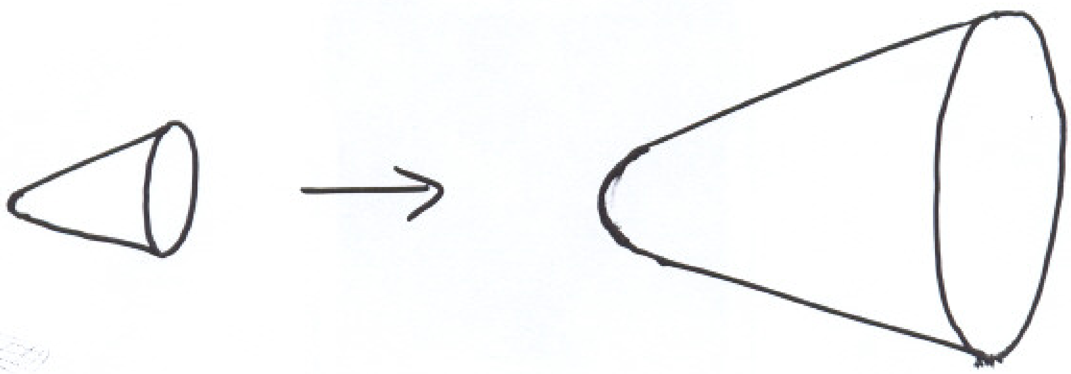
Exact RG flow from $\mathbb{C}/\mathbb{Z}_n$ (UV) to $\mathbb{C}$ (IR) using $\mathcal{N}=2$ GLSM

and Hori-Vafa mirror symmetry

$$\begin{array}{l} \times \ \text{UV free GLSM} \\ \downarrow \\ \times \ \mathbb{C}/\mathbb{Z}_n \\ \downarrow \\ \times \ \mathbb{C} \end{array}$$

Gutperle, MH, Minwalla, Schomerus (hep-th/0211063)

Exact solution to sigma-model β-function equations:



III. (A) $f(r) = \left[1 + W\left((n-1)e^{n-1-r^2/2\alpha'}\right)\right]^{-1}$ $\left(W(x)e^{W(x)} \equiv x\right)$

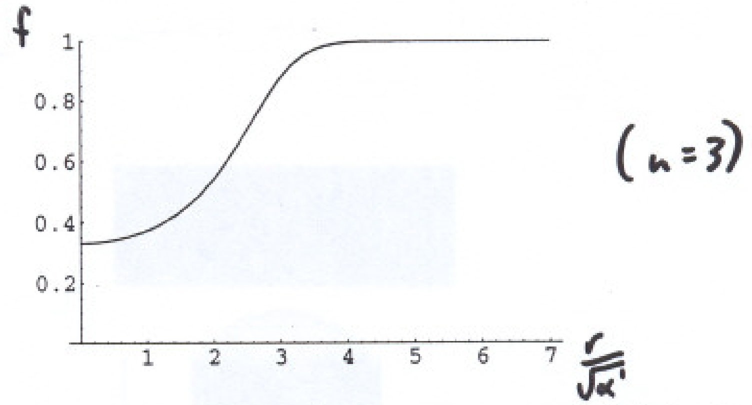


Fig. 3: The function f defined in (A.4), plotted against $r/\sqrt{\alpha'}$, for the case $\zeta = 1/3$.

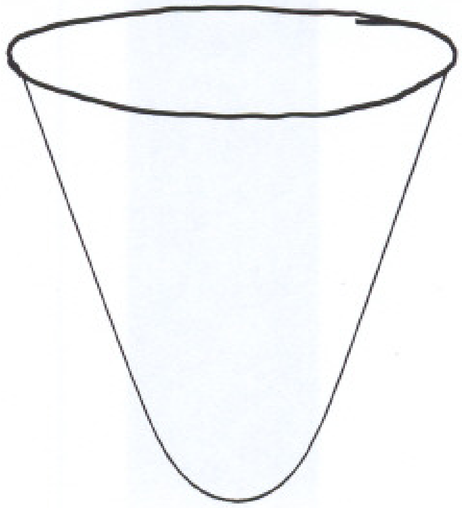


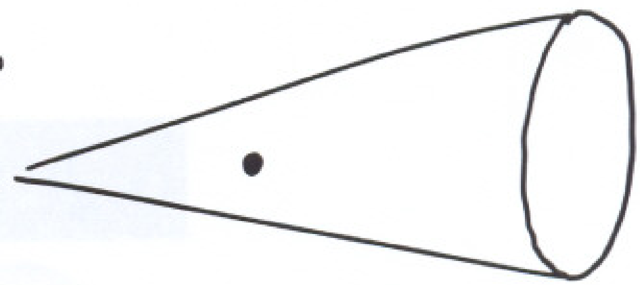
Fig. 4: Cross section of the cone (A.2), (A.3) in the case $\zeta = 1/3$.

eventually diffuses over an infinite area. We can only trust this solution for $\lambda \gg 1$, when the curvature is small enough for eq. (A.1) to be valid. Hence there is no significance to the fact that we cannot continue this solution to negative λ, and as usual, we expect the full RG flow to extend from $\lambda = -\infty$ to $\lambda = \infty$.

The first form (A.2) makes it manifest that the flow proceeds by a global Weyl transformation, i.e. the geometry keeps the same shape while expanding in size. This is analogous to the Gaussian solution of a linear diffusion equation, which over time broadens but always remains a Gaussian. In this sense a Gaussian is a "fixed point modulo broadening" of the diffusion equation, and it is this property which implies that an arbitrary initial distribution (with a finite total amount of matter) will after a sufficiently long time diffuse into a Gaussian distribution. Analogously, since the above solution is a "fixed point modulo global Weyl transformations" of eq. (A.1), we can expect that if at $\lambda \approx 1$ the geometry

D-brane probe

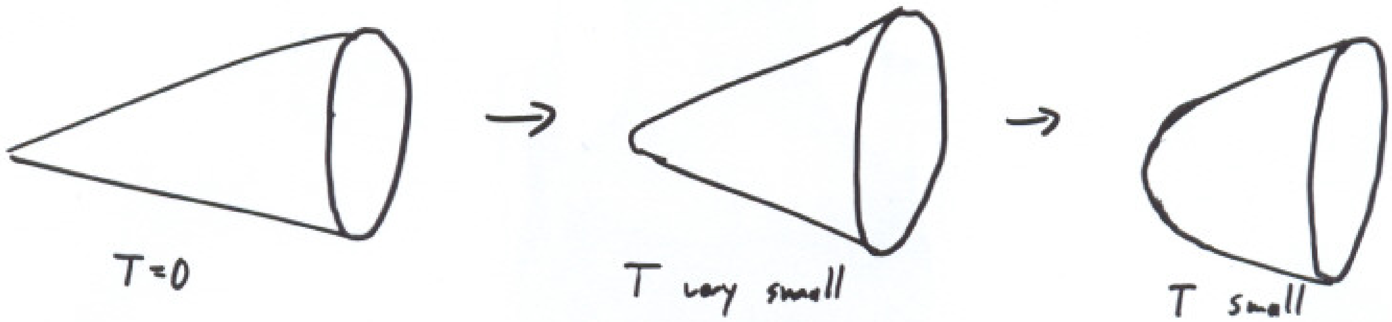
A D-brane localized in $\mathbb{C}/\mathbb{Z}_n$



has a moduli space which is also $\mathbb{C}/\mathbb{Z}_n$

APS + Harvey, Kutasov, Martinec, Moore (hep-th/0111054):

turning on the tachyon deforms the moduli space:



II. Gravity phase: exact solutions

refs,
Gregory + Harvey (hep-th/0306146), MH (hep-th/0312213)

In the dynamical decay of $\mathbb{C}/\mathbb{Z}_n$, the energy must eventually be carried away by massless closed string radiation.

What fields are active? $g_{\mu\nu}$, $\cancel{B_{\mu\nu}}$, Φ

carries away energy

Rotational symmetry yields vast simplification: no gravitational backreaction on Φ EOM!

Einstein equation: $R_{\mu\nu} = 4\,\partial_\mu \Phi\, \partial_\nu \Phi$

Choose light-cone coordinates u, v:

in $\mathbb{C}/\mathbb{Z}_n$: $\sigma = \ln n$

$$ds^2 = -4e^{2\sigma(u,v)}du\,dv + r^2(u,v)\,d\theta^2$$

$$R_{\theta\theta} = 0 \quad \Rightarrow \partial_u \partial_v r = 0 \Rightarrow r(u,v) = r_u(u) + r_v(v)$$

coordinate redefinition: $u = r_u$, $v = -r_v \Rightarrow r = u - v$

$t = u + v$

$$\Rightarrow ds^2 = e^{2\sigma(t,r)}(-dt^2 + dr^2) + r^2\, d\theta^2$$

Φ EOM: $\quad \Box\Phi = \left(-\partial_t^2 + \frac{1}{r}\partial_r r \partial_r\right)\Phi = 0$

independent of σ, same as flat space!

$\Rightarrow$ no gravitational back-reaction

Kinetic energy of Φ sources $\sigma \Rightarrow$ curvature:

$$\partial_t \sigma = 4r\,\partial_t\Phi\,\partial_r\Phi$$

$$\partial_r \sigma = 2r\left[(\partial_t\Phi)^2 + (\partial_r\Phi)^2\right]$$

$\Rightarrow$ apparent horizon + singularity cannot spontaneously form during gravity phase

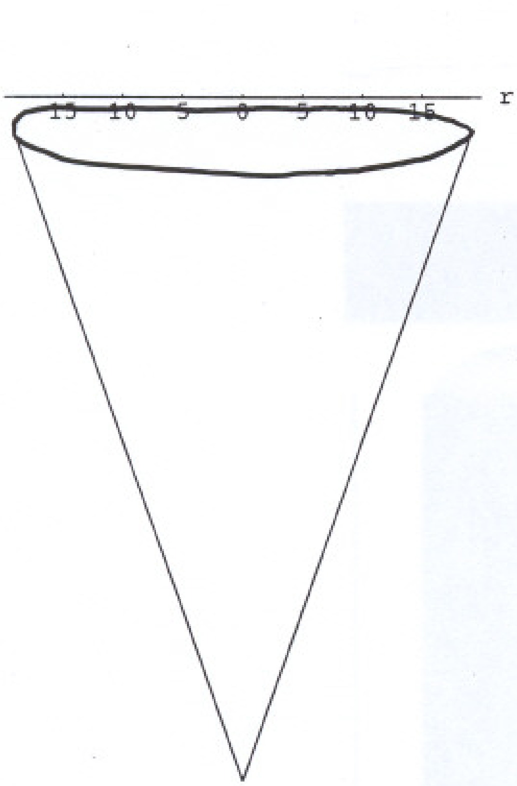
Actually, as energy is dumped into Φ during the decay, we have

$$\Box\Phi = 2\pi\rho(t,r)$$

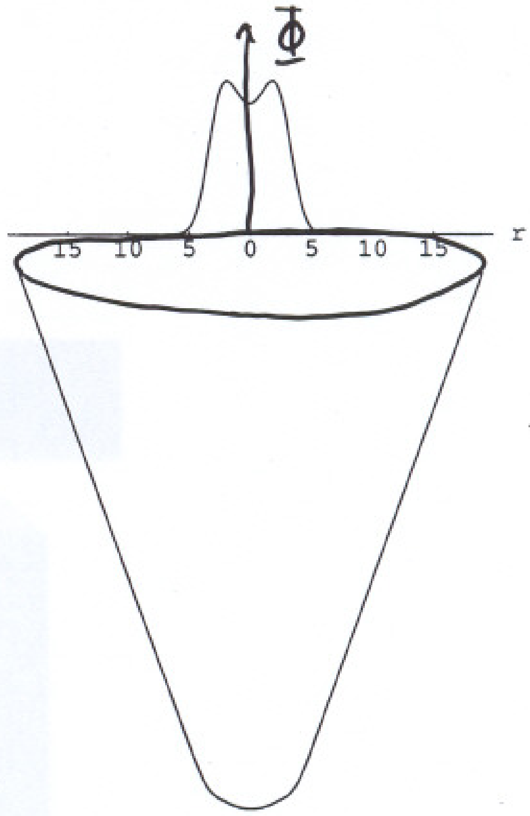
which can be solved using retarded Green function

$$G(t,r) = \begin{cases} 0, & t<r \\ \dfrac{1}{\sqrt{t^2-r^2}}, & t>r \end{cases}$$

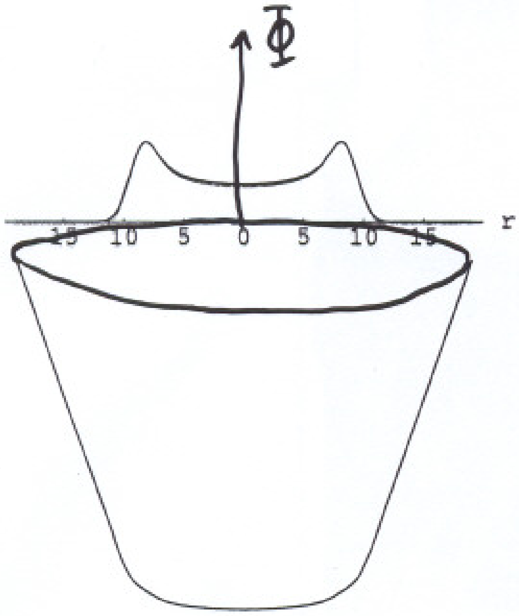
IV. B Numerical example: solve $\Box \Phi = e^{-(r^2+t^2)}$ then find σ



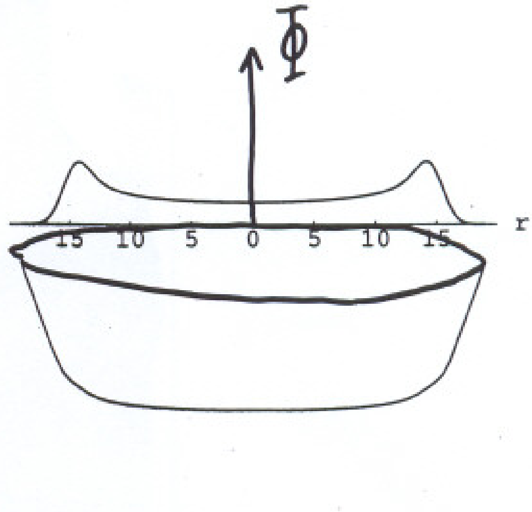
$t = -3$



$t = 3$



$t = 9$



$t = 15$

Change in deficit angle is determined by total energy released.

For example, if ρ is spatially localized

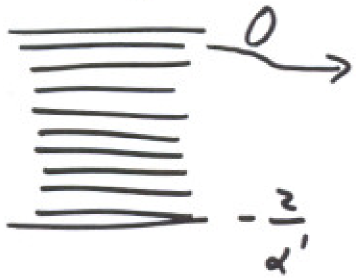
$$\rho(t,r) = \delta^2(r)\,\rho(t)$$

then

$$\ln \frac{n_{final}}{n_{init}} = \Delta\sigma = -2\int_0^\infty d\omega\ \omega\ |\tilde{\rho}(\omega)|^2$$

III. The large n limit

At large n, $\mathbb{C}/\mathbb{Z}_n$ has a dense spectrum of tachyons:



The lightest is in the second twisted sector: $m^2 = -\frac{4}{n\alpha'}$

According to Vafa's picture, condensation of this tachyon leads not to flat space but to $\mathbb{C}/\mathbb{Z}_n$

The energy released in such a decay is small:

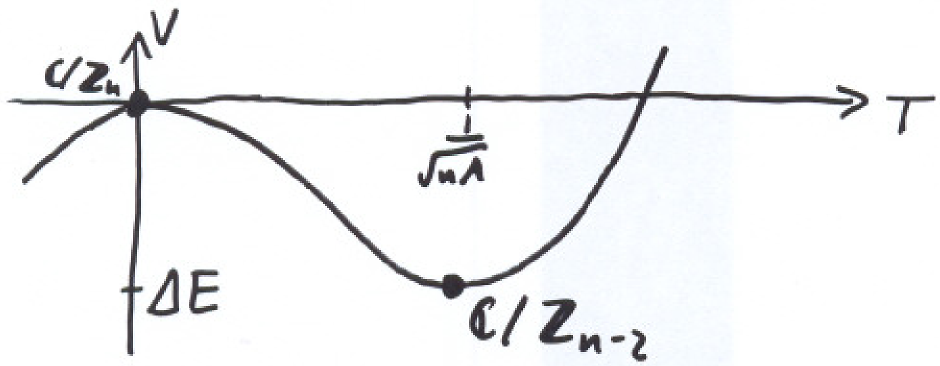
$$\kappa^2 \Delta E = 2\pi\left(1 - \frac{1}{n-2}\right) - 2\pi\left(1 - \frac{1}{n}\right)$$

$$\approx -\frac{4\pi}{n^2}$$

Conjecture: The potential for this tachyon field is turned up by a positive quartic term, and the minimum represents

$$\mathbb{C}/\mathbb{Z}_{n-2}$$

$$\kappa^2 V(T) = -\frac{2}{n} T^2 + A T^4 + \cdots$$



This yields a prediction for A:

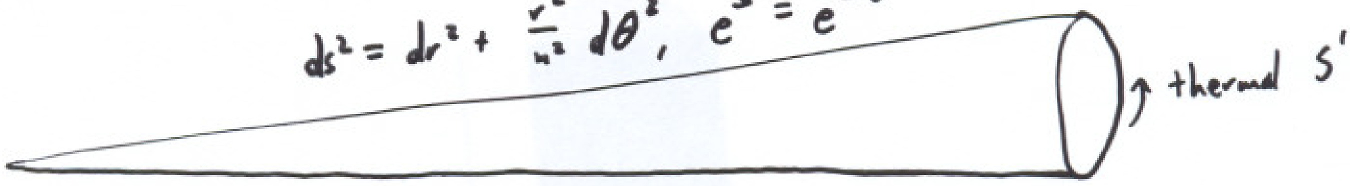
$$A = \frac{1}{4\pi} \quad \text{(no powers of } n\text{)}$$

Dabholkar, Iqubal, Raeymaekers (hep-th/0403238) are working to extract A from on-shell S-matrix elements

APS + Adams, Dabholkar, MH, Raeymaekers (work in progress)

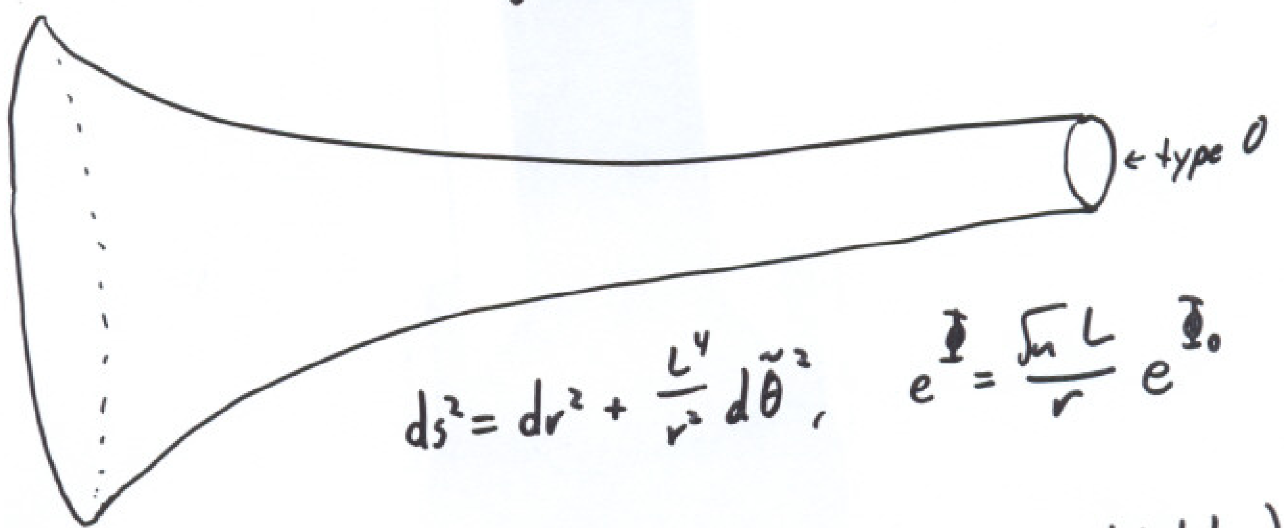
At large n $\mathbb{C}/\mathbb{Z}_n$ has a dual description which elucidates the physics of this light tachyon

$$ds^2 = dr^2 + \frac{r^2}{n^2} d\theta^2, \quad e^{\Phi} = e^{\Phi_0}$$



$r = \quad \sqrt{\alpha'} \qquad \sqrt{n\alpha'} \equiv L \qquad n\sqrt{\alpha'}$

T-duality on θ



$$ds^2 = dr^2 + \frac{L^4}{r^2} d\tilde{\theta}^2, \quad e^{\Phi} = \frac{\sqrt{n}\, L}{r} e^{\Phi_0}$$

(This gravitational solution has an instability.)

strings with small twist number move easily away from the fixed point. They sit in a shallow harmonic oscillator potential, with wave functions extending out to $\sim \sqrt{n\alpha'} \equiv L$

Example: the strings at the bottom of the spectrum are modes of the type 0 tachyon

$$m^2 = -\frac{2}{\alpha'}\left(1 - \frac{1}{n}\right)$$

type 0 tachyon mass

harmonic oscillator zero-point energy

Another example: the light tachyons are gravity modes:

$$\delta\, ds^2 = T\, e^{2i\tilde{\theta} - r^2/L^2}\left(dr - i\frac{L^2}{r}d\tilde{\theta}\right)^2 + \text{c.c.}$$

Since this mode mainly has support in the region where gravity is trustworthy

$$\sqrt{\alpha'} < r < n\sqrt{\alpha'}$$

we can use gravity to (for example) estimate the quartic coupling A

$$S_{grav} = \frac{1}{2\kappa^2}\int d^{10}x \sqrt{G}\, e^{-2\overset{\overset{= \Phi - \Phi_0}{\downarrow}}{\tilde{\Phi}}}\left(R + \cdots\right)$$

$$= \frac{1}{2\kappa^2}\int d^8x \,\frac{1}{n}\,\underbrace{\int dr\, d\theta \sqrt{G} \cdots}_{\substack{\text{dimensionless function of } T,\ \partial_\mu T,\ \ldots \\ \text{depending only on } L,\ \text{not } n \text{ or } \alpha'}}$$

$$\Longrightarrow \kappa^2 V(T) = -\frac{2}{n}T^2 + \frac{\mathcal{O}(1)}{n}T^4 + \cdots$$

In other words, $\frac{1}{n}$ is not a parameter in the gravity background (except as an irrelevant dilaton shift) or EOM, so there cannot be another solution with $T \sim \frac{1}{\sqrt{n}}$.

What is going on in the dangerous region $r < \sqrt{\alpha'}$?

Here Buscher T-duality breaks down, and the correct exact description is given by Hori-Vafa mirror symmetry.

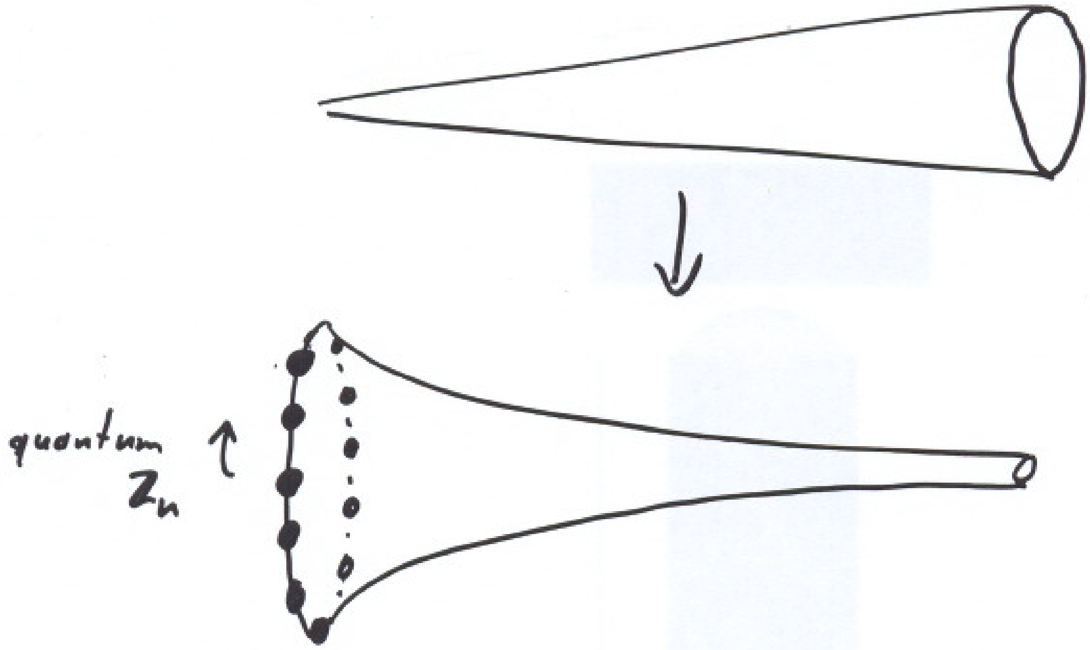
The result is not a sigma-model but rather a Landau-Ginzburg theory with canonical variable

$$U = \frac{r^2}{2n\alpha'} - i\tilde{\theta}$$

and superpotential $W = e^{-nU}$

(Hori, Kapustin hep-th/0104202, Vafa hep-th/0111051)

Rough qualitative picture



D-branes can end on these defects: