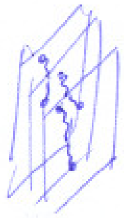
RASTELLI

LECTURES ON OPEN/CLOSED DUALITY – TALK 1
(SMALL COMMENTS)

ROUTES TO GAUGE TH/GEOMETRY CORRESPONDENCE



N

- Effect on spacetime or
- GAUGE TH ON BRANE

} VIEW BRANE AS THESE

- LARGE N → 't Hooft's idea

V vertices

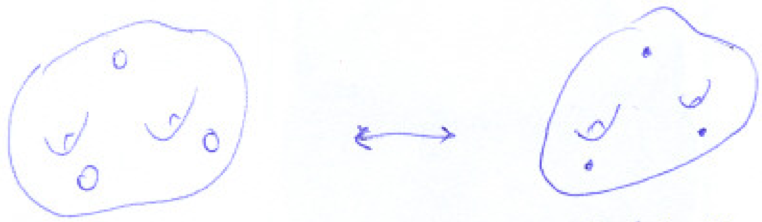
P propagators

h holes

$$(g_{YM}^2)^{P-V} N^h \longrightarrow t = g_{YM}^2 N$$

$$\chi = 2-2g = V-P+h$$

(SURFACE EULER #)



Open case (holes)

$g=2, h=3$

Closed case (no holes)

$g=2$, punctures.

Moduli $= -6+6g+3h$

$$\# \text{Moduli} = -6+6g+2\overset{\substack{\#\text{punctures}\\ = \\ (\#\text{holes})\\ \downarrow}}{p}$$

Lost info is the 'SIZE' of the holes.

⇓

INTEGRATE OVER SIZE OF HOLES

$$t\, b_0 \int d\rho\, \rho^{L_0} |B\rangle_\rho \leftrightarrow t\, w(p)$$

hole size integral

$$\sum_n \frac{t^n}{n!} \prod_{i=1}^{n} \int d\rho_i \longrightarrow \sum_n \frac{t^n}{n!} \left(\int d^2z\, w\right)^n = \exp\left[t \int d^2z\, w\right]$$

Indistinguishability of holes

$$S^{WS} \longrightarrow S^{WS} + \int d^2z\, w \qquad \text{in CFT}$$

Matrix Model

CONSIDER HERMITIAN $N \times N$ MATRICES X, Z

$$S[X] \sim \frac{1}{g_0^2} \mathrm{Tr}\left[ZX^2 + X^3\right] \qquad \underset{\text{DIAGONALIZED}}{Z =} \begin{pmatrix} z_1 & & \\ & \ddots & \\ & & z_N \end{pmatrix}$$

COUPLING CONSTANTS ENCODE OPEN STRING BOUNDARY CONDITIONS $\{z_i\}, i = 1 \dots N$

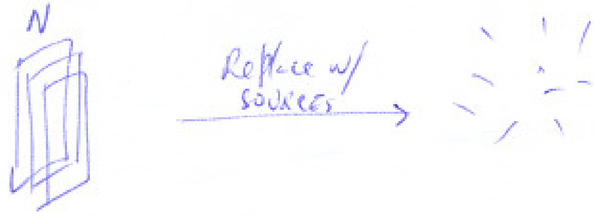
STATEMENT OF OPEN/CLOSED DUALITY

→ CLOSED DIAG. WITH $\underbrace{h \text{ PUNCTURES}}_{= \text{Turning on SOURCES}}$ CAN BE CALCULATED AS OPEN DIAG. WITH HOLES

- NORMALLY THE OPEN CASE IS VASTLY SIMPLER TO CALCULATE
- A CHECK: ∃ ISOMORPHISM $\mathcal{M}^{open}_{g,h} \simeq \mathbb{R}_+^h \times \mathcal{M}^{closed}_{g,p=h}$ (PENNER, KONTSEVICH)

(Moduli space of open surface of g, h) (Moduli space of closed surface of g, p)

SUMMARY OF IDEA: MAKE CORRESPONDENCE PRECISE AT PERTURBATIVE LEVEL



$\mathcal{M}^{open}_{g,h}$

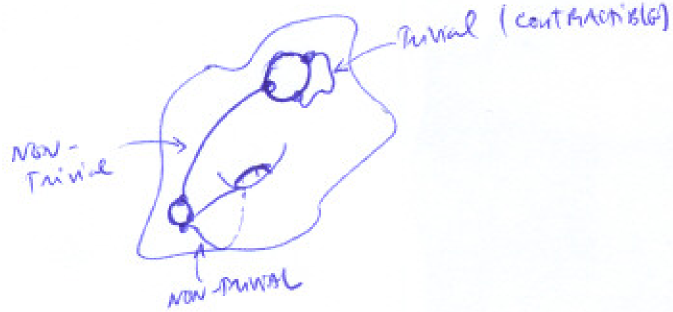
Metric $\longrightarrow$ Complex structure

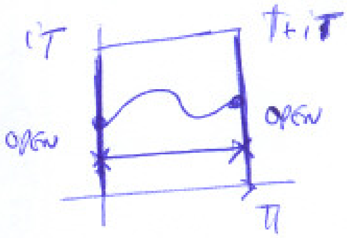
Equivalence class of metrics $\longleftarrow$ Complex structure

If we want

complex str $\longrightarrow$ metric, we must supply something else

Given a complex structure, look for the metric of minimal area such that any non-trivial open curve has $L_\gamma \geq \pi$ (lenght $\geq \pi$)

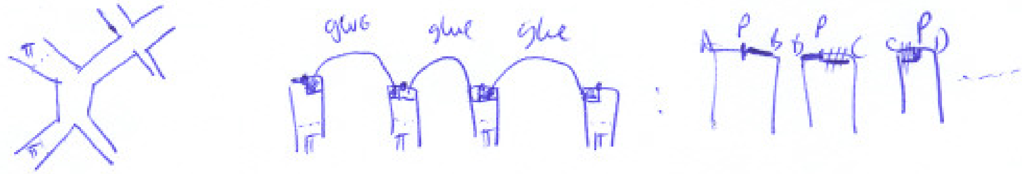




$ds^2 = dz\,d\bar{z}$ (flat)

This gives a preferred metric $\cong$ complex str.

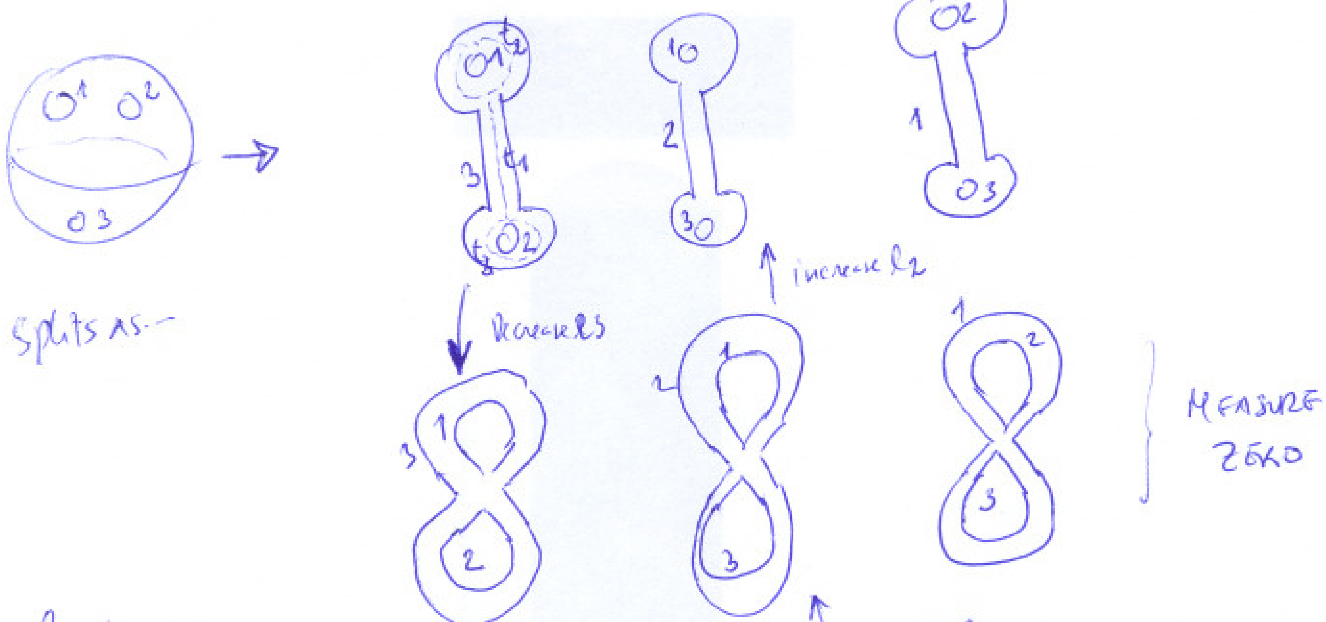
Locally, a general metric should look like strips of lenght π.



Flat strips that meet at a conical singularity!

$\dim = -6 + 6g + 3h$. EXAMPLE: $\{t_i\}$ AS MODULI

$g = 0,\ h = 3 \quad \dim = 3$

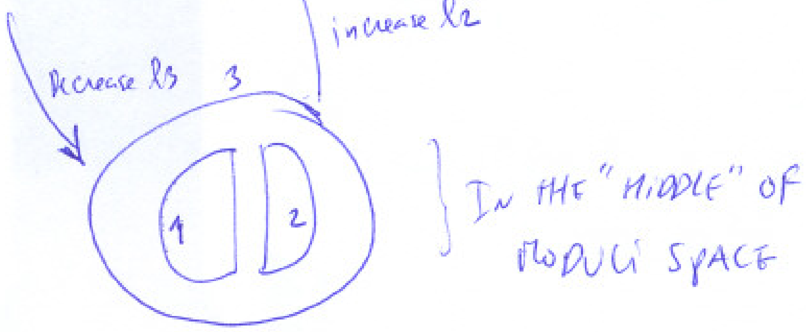


l_i: lenght of BOUNDARY i

$l_1 = t_2$

$l_2 = t_3$

$l_3 = t_1 + t_2 + t_3$



STRING FIELD THEORY ACTION → $S[\psi] = \frac{1}{2}\langle\psi, c_0 L_0 \psi\rangle + \frac{1}{3}\langle\psi,\psi,\psi\rangle$

$\frac{b_0}{L_0} = \int_0^\infty e^{-tL_0}\, dt$

t : L_0 PROPAGATES ON t

NO QUARTIC VERTEX, WHICH CAN BE SEEN AS SUBSET OF CUBIC VERTEX