On Higher Spins

Part II

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(Based on: *Francia, AS, hep-th/0207002, hep-th/0212185; AS, Tsulaia, hep-th/0311257; AS, Sezgin,Sundell, to appear)*

Modern Trands in String Theory II
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Summary

- Part I:
  - From Fierz-Pauli to Fronsdal
  - Free (non-local) geometric equations
- Part II:
  - Triplets and (local) compensator form
  - Links with the Vasiliev equations

Geometric equations

1. Odd spins (s=2n+1):

$$\partial_\mu F^{\mu\nu} = 0 \quad \Rightarrow \quad \frac{1}{\Box^n}\partial_\mu R^{[n]\mu;\nu_1\ldots\nu_s} = 0$$

2. Even spins (s=2n):

$$R^{\mu\nu} = 0 \quad \Rightarrow \quad \frac{1}{\Box^{n-1}} R^{[n];\nu_1\ldots\nu_s} = 0$$

Fermions

Notice:

$$\mathcal{S}_{\left(s+\frac{1}{2}\right)} - \frac{1}{2}\frac{\partial}{\Box}\,\partial\!\!\!/\;\mathcal{S}\!\!\!/_{\left(s+\frac{1}{2}\right)} = \frac{i\partial\!\!\!/}{\Box}\mathcal{F}_{(s)}$$

Example: spin 3/2 (Rarita-Schwinger)

$$\gamma^{\mu\nu\rho}\partial_\nu\psi_\rho = 0$$

$$\mathcal{S}_\mu \equiv i\left(\partial\!\!\!/\,\psi_\mu - \partial_\mu\psi\!\!\!/\right) = 0$$

$$\mathcal{S}_\mu - \frac{1}{2}\frac{\partial_\mu}{\Box}\,\partial\!\!\!/\;\mathcal{S}\!\!\!/ = \frac{i\partial\!\!\!/}{\Box}\left[\Box\eta_{\mu\nu} - \partial_\mu\partial_\nu\right]\psi^\nu$$

Fermions

One can again iterate:

$$\mathcal{S}^{(n+1)} = \mathcal{S}^{(n)} + \frac{1}{n(2n+1)} \frac{\partial^{\,2}}{\Box} \mathcal{S}^{(n)\,\prime} - \frac{2}{2n+1} \frac{\partial}{\Box} \partial\cdot\mathcal{S}^{(n)}$$

The relation to bosons generalizes to:

$$\mathcal{S}^{(n)}_{s+1/2} - \frac{1}{2n} \frac{\partial}{\Box} \not{\partial} \not{\mathcal{S}}^{(n)}_{s+1/2} = i\, \frac{\not{\partial}}{\Box} \mathcal{F}^{(n)}_{s}(\psi)$$

The Bianchi identity generalizes to:

$$\partial\cdot\mathcal{S}^{(n)} - \frac{1}{2n} \partial\, \mathcal{S}^{(n)\,\prime} - \frac{1}{2n} \not{\partial} \not{\mathcal{S}}^{(n)} = i\, \frac{\partial^{\,2n}}{\Box^{\,n-1}} \not{\psi}^{[n]}$$

Bosonic string: BRST

- The starting point is the Virasoro algebra:

$$L_k = \frac{1}{2} \sum_{l=-\infty}^{+\infty} \alpha^{\mu}_{k-l}\, \alpha_{\mu l}$$

$$[L_k, L_l] = (k-l)\, L_{k+l} + \frac{\mathcal{D}}{12}\, m\,(m^2 - 1)$$

- In the tensionless limit, one is left with:

$$\ell_0 = p^2$$
$$\ell_k = p \cdot \alpha_k$$

- Virasoro contracts (no c. charge):

$$[\ell_k, \ell_l] = k\, \delta_{k+l,\,0}\ \ell_0$$

String Field equation

Higher-spin massive modes:

- massless for $1/\alpha' \rightarrow 0$
- Free dynamics can be encoded in:

$$Q|\psi\rangle = 0 \qquad (Q^2 = 0)$$
$$\delta|\psi\rangle = Q|\Lambda\rangle$$

(Kato and Ogawa, 1982)
(Witten, 1985)
(Neveu, West et al, 1985)

NO trace constraints on φ or Λ

Low-tension limit

- Similar simplifications for BRST charge:

$$\mathcal{Q} = \sum_{-\infty}^{+\infty} \left[C_{-k} L_k - \frac{1}{2}(k-l) : C_{-k} C_{-l} B_{k+l} : \right] - C_0$$

$$Q = \sum_{-\infty}^{+\infty} \left[c_{-k} \ell_k - \frac{k}{2} b_0 \, c_{-k} c_k \right] \quad (Q^2 = 0 \quad \forall \mathcal{D})$$

- Making zero-modes manifest:

$$Q = c_0 \, \ell_0 - b_0 \, M + \tilde{Q}$$

$$|\Phi\rangle = |\varphi_1\rangle + c_0 \, |\varphi_2\rangle$$

$$|\Lambda\rangle = |\Lambda_1\rangle + c_0 \, |\Lambda_2\rangle$$

$$\tilde{Q} = \sum_{k \neq 0} c_{-k} \, \ell_k$$

$$M = \frac{1}{2} \sum_{-\infty}^{+\infty} k \; c_{-k} \, c_k$$

Symmetric triplets

(A. Bengtsson, 1986)
(Henneaux,Teitelboim, 1987)
(Pashnev, Tsulaia, 1998)
(Francia, AS, 2002)

- Emerge from $\alpha_{-1}, b_{-1}, c_{-1}$

$$
\begin{aligned}
|\varphi_1\rangle &= \frac{1}{s!}\,\varphi_{\mu_1\ldots\mu_s}(x)\alpha_{-1}^{\mu_1}\ldots\alpha_{-1}^{\mu_s}\,|0\rangle \\
&\quad + \frac{1}{(s-2)!}\,D_{\mu_1\ldots\mu_{s-2}}(x)\alpha_{-1}^{\mu_1}\ldots\alpha_{-1}^{\mu_{s-2}}\,c_{-1}\,b_{-1}\,|0\rangle \\
|\varphi_2\rangle &= \frac{-i}{(s-1)!}\,C_{\mu_1\ldots\mu_{s-1}}(x)\alpha_{-1}^{\mu_1}\ldots\alpha_{-1}^{\mu_{s-1}}\,b_{-1}\,|0\rangle \\
|\Lambda\rangle &= \frac{i}{(s-1)!}\,\Lambda_{\mu_1\mu_2\ldots\mu_{s-1}}(x)\,\alpha_{-1}^{\mu_1}\ldots\alpha_{-1}^{\mu_{s-1}}b_{-1}\,|0\rangle
\end{aligned}
$$

- **The triplets are:**

$$
\begin{aligned}
\Box\,\varphi &= \partial C\,, \\
\partial\cdot\varphi - \partial D &= C \\
\Box\,D &= \partial\cdot C
\end{aligned}
\qquad
\begin{aligned}
\delta\varphi &= \partial\Lambda\,, \\
\delta C &= \Box\,\Lambda\,, \\
\delta D &= \partial\cdot\Lambda
\end{aligned}
$$

Symmetric triplets

- Can also eliminate C:

$$\mathcal{F} = \partial^2 \left(\varphi' - 2D\right)$$
$$\Box\, D = \frac{1}{2}\partial\cdot\partial\cdot\varphi - \frac{1}{2}\partial\,\partial\cdot D\,,$$

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu\varphi)^2 + s\,\partial\cdot\varphi\, C + s(s-1)\,\partial\cdot C\, D + \frac{s(s-1)}{2}(\partial_\mu D)^2 - \frac{s}{2}C^2\,,$$
$$\mathcal{L} = -\frac{1}{2}(\partial_\mu\varphi)^2 + \frac{s}{2}(\partial\cdot\varphi)^2 + s(s-1)\,\partial\cdot\partial\cdot\varphi\, D + s(s-1)\,(\partial_\mu D)^2 + \frac{s(s-1)(s-2)}{2}(\partial\cdot D)^2$$

- Gauge theories of $\ell_0,\ \ell_{\pm 1}$

- **Physical state conditions:**
- Propagate spins s,s-2, …, 0 or 1

$$\Box\varphi = 0$$
$$\partial\cdot\varphi = 0$$
$$\left[\varphi' = 0\right]$$

(A)dS symmetric triplets

- Can build directly, deforming flat-space triplets, or via BRST
  - **Directly:** insist on relation between C and others
  - **BRST:** gauge non-linear constraint algebra
  - Basic commutator: $[\nabla_\mu, \nabla_\nu]\, V_\rho \;=\; \frac{1}{L^2}\,(g_{\nu\rho} V_\mu \;-\; g_{\mu\rho} V_\nu)$

$$\delta\varphi = \nabla\Lambda$$
$$\delta D = \nabla\cdot\Lambda$$
$$\delta C = \Box\,\Lambda + \frac{(s-1)(3-s-\mathcal{D})}{L^2}\,\Lambda \;+\; \frac{2}{L^2}\, g\,\Lambda'$$

$$\Box\,\varphi = \nabla C + \frac{1}{L^2}\left\{8\,g\,D - 2\,g\,\varphi' \;+\; [(2-s)(3-\mathcal{D}-s) - s]\,\varphi\right\}$$
$$C = \nabla\cdot\varphi - \nabla D$$
$$\Box\,D = \nabla\cdot C + \frac{1}{L^2}\left\{[s(\mathcal{D}+s-2)+6]D - 4\varphi' - 2gD'\right\}$$

(A)dS symmetric triplets

- Can also deform directly the equations without C:

$$\mathcal{F} = \frac{1}{2}\{\nabla,\nabla\}\left(\varphi' - 2D\right) + \frac{1}{L^2}\{8\,g\,D \;-\; 2\,g\,\varphi' + \;[(2-s)(3-\mathcal{D}-s)-s]\;\varphi\}$$

$$\Box\,D \;+\; \frac{1}{2}\,\nabla\nabla\cdot D - \frac{1}{2}\,\nabla\cdot\nabla\cdot\varphi = -\;\frac{(s-2)(4-\mathcal{D}-s)}{2L^2}\;D \;-\; \frac{1}{L^2}\,g\;D' + \frac{1}{2L^2}\left\{[s(\mathcal{D}+s-2)+6]D - 4\varphi' - 2gD'\right\}$$

- It is convenient to define $\mathcal{F}_L \;=\; \mathcal{F} - \frac{1}{L^2}\left\{[(3-\mathcal{D}-s)(2-s)-s]\;\varphi \;+\; 2\,g\,\varphi'\right\}$

- Bianchi identity and first equation then become:

$$\nabla\cdot\mathcal{F}_L \;-\; \frac{1}{2}\nabla\,\mathcal{F}_L' \;=\; -\;\frac{3}{2}\;\nabla^3\;\varphi'' \;+\; \frac{2}{L^2}\;g\;\nabla\,\varphi''$$

$$\mathcal{F}_L = \frac{1}{2}\{\nabla,\nabla\}\left(\varphi' - 2D\right) + \frac{8}{L^2}\;g\;D$$

(A)dS symmetric triplets

- The deformed equations can be derived from

$$
\begin{aligned}
\mathcal{L} \;=\; & -\frac{1}{2}\,(\nabla_\mu \varphi)^2 \;+\; s\,\nabla\cdot\varphi\, C \;+\; s(s-1)\,\nabla\cdot C\, D \;+\; \frac{s(s-1)}{2}\,(\nabla_\mu D)^2 \;-\; \frac{s}{2}\,C^2 \\
& +\; \frac{s(s-1)}{2L^2}\,(\varphi')^2 \;-\; \frac{s(s-1)(s-2)(s-3)}{2L^2}\,(D')^2 \;-\; \frac{4s(s-1)}{L^2}\,D\,\varphi' \\
& -\; \frac{1}{2L^2}\,[(s-2)(\mathcal{D}+s-3) \;-\; s]\,\varphi^2 \;+\; \frac{s(s-1)}{2L^2}\,[s(\mathcal{D}+s-2)+6]\;D^2
\end{aligned}
$$

- Alternatively:
  - modify momenta in contracted Virasoro
  - gauge algebra → **non linear** (but M,N diagonal on triplets)

$$
\ell_0 \rightarrow p^a\, p_a \;-\; i\,\omega_a{}^{ab}\, p_b
$$

$$
[\ell_1, \ell_{-1}] = \ell_0 - \frac{1}{L^2}\left(-\mathcal{D} + \frac{\mathcal{D}^2}{4} + 4\, M^\dagger\, M - N^2 + 2\, N\right)
$$

$$
N \;=\; \alpha_{-1}\cdot\alpha_1 \;+\; \frac{\mathcal{D}}{2}\;, \qquad M = \frac{1}{2}\,\alpha_1\cdot\alpha_1
$$

Compensator Equations

- In the triplet:

$$\mathcal{F} = \partial^2 \left(\varphi' - 2D\right)$$
$$\Box\, D = \frac{1}{2}\partial \cdot \partial \cdot \varphi - \frac{1}{2}\partial\, \partial \cdot D$$

- spin-(s-3) **compensator**:

$$\varphi' - 2\,D = \partial\,\alpha$$

- The first becomes:

$$\mathcal{F} = 3\,\partial^{\,3}\,\alpha$$

- The second becomes:

$$\mathcal{F}' - \partial^2\,\varphi'' = 3\,\Box\partial\alpha + 2\,\partial^2\,\partial \cdot \alpha$$

- Combining them:

$$\partial^2\,\varphi'' = 4\,\partial^2\,\partial\cdot\alpha + \partial^3\,\alpha' = \partial^2\left(4\partial \cdot \alpha + \partial\alpha'\right)$$

- Finally (also from Bianchi):

$$\varphi'' = 4\partial \cdot \alpha + \partial\alpha'$$

Compensator Equations

- Summarizing:

$$\mathcal{F} = 3\,\partial^{3}\,\alpha$$
$$\varphi'' = 4\partial\cdot\alpha + \partial\alpha'$$
$$\delta\varphi = \partial\Lambda$$
$$\delta\alpha = \Lambda'$$

- Describe a spin-s gauge field with:
  - **NO** trace constraints on the gauge parameter
  - **NO** trace constraints on the gauge field
  - First can be reduced to minimal non-local form
- **BUT:**
  - **NOT** Lagrangian equations

(A)dS Compensator Eqs

- Flat-space compensator equations can be extended to (A)dS:

$$\mathcal{F} = 3\nabla^3\alpha + \frac{1}{L^2}\{[(3-\mathcal{D}-s)(2-s)-s]\varphi + 2g\varphi'\} - \frac{4}{L^2}\, g\nabla\alpha$$
$$\varphi'' = 4\nabla\cdot\alpha + \nabla\alpha'$$

  - Gauge invariant under

$$\delta\,\varphi = \nabla\Lambda\,, \quad \delta\,\alpha = \Lambda'$$

- The first can be turned into the second via (A)dS Bianchi

$$\nabla\cdot\mathcal{F}_L - \frac{1}{2}\nabla\mathcal{F}_L' = -\frac{3}{2}\nabla^3\varphi'' + \frac{2}{L^2}\, g\,\nabla\varphi''$$
$$\mathcal{F}_L \equiv \mathcal{F} - \frac{1}{L^2}\left\{[(3-\mathcal{D}-s)(2-s)-s]\,\varphi + 2\,g\,\varphi'\right\}$$

Compensator Equations

- It is possible to obtain a Lagrangian form of the compensator equations, using BRST techniques
- **Essentially in Pashnev and Tsulaia (1997)**
- Formulation involves number of fields ~ s
- Interesting BRST subtleties
- Here we discuss explicitly spin s=3

- Fields:

$$\varphi, C, D, \alpha, \left[\varphi^{(1)}, C^{(1)}, E, F\right]$$
$$\Lambda, \left[\Lambda^{(1)}, \mu\right]$$

Compensator Equations

- Gauge transformations:

$$\delta\varphi = \partial\Lambda + \eta\mu \qquad \delta\alpha = \Lambda' - \sqrt{2\mathcal{D}}\Lambda^{(1)}$$
$$\delta\varphi^{(1)} = \partial\Lambda^{(1)} + \sqrt{\tfrac{\mathcal{D}}{2}}\mu \qquad \delta C = \Box\Lambda$$
$$\delta D = \partial\cdot\Lambda + \mu \qquad \delta C^{(1)} = \Box\Lambda^{(1)}$$
$$\delta E = \partial\cdot\mu \qquad \delta F = \Box\mu$$

- Field equations:

$$\Box\varphi = \partial C + \eta F \qquad \Box\alpha = C' - \sqrt{2\mathcal{D}}C^{(1)}$$
$$\partial\cdot\varphi - \partial D - \eta E = C \qquad \Box\varphi^{(1)} = \partial C^{(1)} + \sqrt{\frac{\mathcal{D}}{2}}F$$
$$\Box D = \partial\cdot C + F \qquad \Box E = \partial\cdot F$$
$$\partial\alpha = \varphi' - 2D - \sqrt{2\mathcal{D}}\varphi^{(1)} \qquad \partial\cdot\varphi^{(1)} - \sqrt{\mathcal{D}2}E = C^{(1)}$$

- Gauge fixing: $\mu : \varphi^{(1)} \to 0 \,, \quad \Lambda^{(1)} : C^{(1)} \to 0$

- Other extra fields: zero by field equations → φ, C, D, α

Fermionic Triplets

(Francia and AS, 2002)

- Counterparts of bosonic triplets
- **GSO:** not in 10D susy strings
- **Yes:** mixed sym generalizations
- Enter directly **type-0 models**

$$
\begin{aligned}
\partial\!\!\!/ \psi &= \partial \chi \\
\partial \cdot \psi - \partial \lambda &= \partial\!\!\!/ \chi \\
\partial\!\!\!/ \lambda &= \partial \cdot \chi \\
\delta \psi &= \partial \epsilon \\
\delta \Lambda &= \partial \cdot \epsilon \\
\delta \chi &= \partial\!\!\!/ \epsilon
\end{aligned}
$$

- Propagate s+1/2 and **all** lower ½-integer spins

Fermionic Compensators

- Recall:

$$\mathcal{S} \equiv i\,(\not\partial\psi - \partial\,\not\psi)$$

$$\partial\cdot\mathcal{S} \;-\; \frac{1}{2}\partial\,\mathcal{S}' \;-\; \frac{1}{2}\not\partial\,\not\mathcal{S} \;=\; i\,\partial^2\,\not\psi'$$

- Spin-(s-2) compensator:

$$\mathcal{S} = -2\,i\,\partial^2\,\xi\ ,$$

$$\not\psi' = 2\,\partial\cdot\xi \;+\; \partial\,\xi' + \;\not\partial\,\not\xi$$

- Gauge transformations:

$$\delta\psi = \partial\,\epsilon\ ,$$

$$\delta\xi = \not\epsilon$$

First compensator equation → second via Bianchi

Fermionic Compensators

- We **could** extend the fermionic compensator eqs to (A)dS
- We **could not** extend the fermionic triplets
- **BRST:** operator extension does not define a closed algebra

$$
\begin{aligned}
(\not\nabla\,\psi - \nabla\,\not\psi) + \frac{1}{2L}\,[\mathcal{D} + 2(n-2)]\,\psi + \frac{1}{2L}\gamma\,\not\psi
&= -\{\nabla,\nabla\}\xi + \frac{1}{L}\gamma\,\nabla\,\xi + \frac{3}{2L^2}\,g\,\xi\ , \\
\not\psi\,' = 2\,\nabla\cdot\xi + \not\nabla\,\not\xi + \nabla\xi\,' + \frac{1}{2L}\,[\mathcal{D} + 2(n-2)]\,\not\xi - \frac{1}{2L}\gamma\,\xi\,' & \\
\mathcal{S} = i\,(\not\nabla\,\psi - \nabla\,\not\psi) + \frac{i}{2L}\,[\mathcal{D} + 2(n-2)]\,\psi + \frac{i}{2L}\gamma\,\not\psi &
\end{aligned}
$$

$$
\begin{aligned}
\delta\psi &= \nabla\,\epsilon + \frac{1}{2L}\gamma\epsilon \\
\delta\xi &= \not\epsilon
\end{aligned}
$$

- First compensator equation → second via (A)dS Bianchi identity:

$$
\begin{aligned}
\nabla\cdot\mathcal{S} - \frac{1}{2}\nabla\,\mathcal{S}\,' - \frac{1}{2}\,\not\nabla\,\not\mathcal{S}
&= \frac{i}{4L}\gamma\,\mathcal{S}\,' + \frac{i}{4L}\,[(\mathcal{D}-2) + 2\,(n-1)]\,\not\mathcal{S} \\
&+ \frac{i}{2}\left[\{\nabla,\nabla\} - \frac{1}{L}\,\gamma\,\nabla - \frac{3}{2\,L^2}\right]\not\psi\,'
\end{aligned}
$$

The Vasiliev equations

(Vasiliev, 1991-2003;Sezgin,Sundell, 1998-2003)

- **Integrable curvature constraints** on one-forms and zero-forms
  - **Conceptual basis:** Cartan integrable systems *(D'Auria,Fre', 1983)*
  - **Key new addition of Vasiliev:** twisted-adjoint representation

- **Minimal case (only symmetric tensors of even rank)**:
  - **zero-form Φ :** Weyl curvatures
  - **one-form A :** gauge fields
  - gauge generators → oscillator star-algebra → **Sp(2,R)**

$$[Y_{iA}, Y_{jB}] = 2i\epsilon_{ij}\ \eta_{AB}$$

$$\widehat{f}(Z;Y)\star\widehat{g}(Z;Y) = \int \frac{d^{2(D+1)}S\, d^{2(D+1)}T}{(2\pi)^{2(D+1)}}\, \widehat{f}(Z+S;Y+S)\, \widehat{g}(Z-T;Y+T)\, e^{iT^{iA}S_{iA}}$$

$$(\widehat{f})^\dagger = \widehat{\bar{f}}(-Z;Y)\ ,\ \tau(\widehat{f}) = \widehat{f}(-iZ;iY)\ ,\ \pi(\widehat{f}) = \widehat{f}(Z^i_a, -Z^i; Y^i_a, -Y^i)$$

$$(\lambda d\widehat{f})^\dagger = \bar{\lambda} d(\widehat{f}^\dagger)\ ,\ \tau(\lambda d\widehat{f}) = \lambda d\tau(\widehat{f})\ ,\ \pi(\lambda d\widehat{f}) = \lambda d\pi(\widehat{f})$$

The Vasiliev equations

- **Curvature constraints:**
  - **[extra non comm. Coords]**

$$
\begin{aligned}
\widehat{F} &= \frac{i}{4} dZ^i \wedge dZ_i \;\kappa \star \widehat{\Phi} \\
\widehat{D}\widehat{\Phi} &= 0 \\
\widehat{D}\widehat{K}_{ij} &= 0 \\
\widehat{K}_{ij} \star \widehat{\Phi} &= \begin{cases} \widehat{\Phi} \star \pi(\widehat{K}_{ij}) & \Rightarrow \text{Off-shell} \\ \mathbf{0} & \Rightarrow \text{On-shell} \end{cases}
\end{aligned}
$$

$$
\begin{aligned}
\widehat{F} &= d\widehat{A} + \widehat{A} \star \wedge \widehat{A} & \tau(\widehat{A}) = (\widehat{A})^\dagger = -\widehat{A} \\
\widehat{D}\widehat{\Phi} &= d\widehat{\Phi} + \widehat{A} \star \widehat{\Phi} - \Phi \star \pi(\widehat{A}) & \tau(\widehat{\Phi}) = (\widehat{\Phi})^\dagger = \pi(\widehat{\Phi}) \\
d &= \underbrace{dX^M \partial_M}_{\text{space time } \mathcal{M}} + \underbrace{dZ^{A\,i} \partial^{(Z)}_{A\,i}}_{\text{internal:BRST-like}} \\
\widehat{A} &= dX^M \widehat{A}_M(X, Z; Y) + dZ^{A\,i} \widehat{A}_{A\,i}(X, Z; Y) & \widehat{\Phi} = \widehat{\Phi}(X, Z; Y)
\end{aligned}
$$

- **Gauge symmetry:**

$$
\begin{aligned}
\delta_{\widehat{\epsilon}} \widehat{A} &= d\widehat{\epsilon} + [\widehat{A}, \widehat{\epsilon}]_\star \\
\delta_{\widehat{\epsilon}} \widehat{\Phi} &= -[\widehat{\epsilon}, \widehat{\Phi}]_\pi \\
[\delta_{\widehat{\epsilon_1}}, \delta_{\widehat{\epsilon_2}}] &= \delta_{[\widehat{\epsilon_1}, \widehat{\epsilon_2}]_\star}
\end{aligned}
$$

The Vasiliev equations

(AS,Sezgin,Sundell, to appear)

- "Off-shell": definitions of Riemann-like curvatures
- "On-shell": Riemann-like = Wey-like l → Ricci-like = 0

- What is the role of Sp(2,R) in this transition?

- Sp(2,R) generators:

$$\begin{aligned} \widehat{K}_{ij} &= \frac{1}{2}\left(Y_i^A Y_{A\,j} - Z_i^A Z_{A\,j} + \widehat{S}_{(i}^A \star \widehat{S}_{j)\,A}\right) = \widehat{K}_{ij}^\dagger = \tau(\widehat{K}_{ij}) \\ \widehat{S}_i^A &= Z_i^A - 2i\widehat{A}_i^A \end{aligned}$$

- Key on-shell constraint:

$$\widehat{K}_{ij} \star \widehat{\Phi} = 0$$

- gauge fields **NOT constrained**

- The strong constraint ensures that the **proper scalar masses** emerge
- At the interaction level → must **regulate projector**

- **Gauge fields: extended (uncostrained) gauge symmetry**

The Vasiliev equations

- **Non-linear corrections:** from dependence on extra coordinates

$$
\begin{aligned}
\widehat{\Phi} &= \Phi - Z^i \int_0^1 dt \left(\widehat{A}_i \star \widehat{\Phi} + \widehat{\Phi} \star \pi(\widehat{A}_i) \right)_{Z \to tZ} \\
\widehat{A}_{ai} &= 0 \quad (\Rightarrow \widehat{M}_{ab} = M_{ab} \text{ below}) \\
\widehat{A}_i &= \underbrace{\partial_i \widehat{\zeta}}_{=0} + Z_i \int_0^1 t dt \left(-i\kappa \star \widehat{\Phi} + \widehat{A}^j \star \widehat{A}_j \right)_{Z \to tZ} \\
\widehat{A}_M &= A_M + Z^i \int_0^1 dt \Big(\underbrace{\partial_M \partial_i \widehat{\zeta}}_{=0} + [\widehat{A}_M, \widehat{A}_i]_\star \Big)_{Z \to tZ}
\end{aligned}
$$

$$
\widehat{A}_M = \frac{1}{1 - \widehat{L}_{(1)} - \widehat{L}_{(2)} - \cdots} A_M \,, \quad \widehat{\Phi} = \frac{1}{1 - \widehat{L}'_{(1)} - \widehat{L}'_{(2)} - \cdots} \Phi
$$

$$
\begin{aligned}
\widehat{L}_{(n)}(\widehat{f}) &= i \int_0^1 \frac{dt}{t} \left(\widehat{A}^i_{(n)} \star \partial_i^{(-)} \widehat{f} + \partial_i^{(+)} \widehat{f} \star \widehat{A}^i_{(n)} \right)_{Z \to tZ} \\
\widehat{L}'_{(n)}(\widehat{f}) &= i \int_0^1 \frac{dt}{t} \left(\widehat{A}^i_{(n)} \star \partial_i^{(-)} \widehat{f} - \partial_i^{(+)} \widehat{f} \star \pi(\widehat{A}^i_{(n)}) \right)_{Z \to tZ} \\
\widehat{A}_{i(n)} &= Z_i \int_0^1 t\, dt \left(-i\kappa \star \widehat{\Phi}_{(n)} + \sum_{m=1}^{n-1} \widehat{A}^j_{(m)} \star \widehat{A}_{j(n-m)} \right)_{Z \to tZ}
\end{aligned}
$$

$$
\widehat{F}_{MN}|_{Z=0} = 0 \,, \quad \widehat{D}_M \widehat{\Phi}|_{Z=0} = 0
$$

$$
\begin{aligned}
F_{MN} &= -\sum_{m=1}^{\infty} \sum_{n+p=m} [\widehat{A}_{(n)M}, \widehat{A}_{(p)N}]_\star |_{Z=0} \\
D_M \Phi &= -\sum_{m=2}^{\infty} \sum_{n+p=m} [\widehat{A}_{(n)M}, \widehat{\Phi}_{(p)}]_\pi |_{Z=0}
\end{aligned}
$$

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