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THERMAL DECAY OF THE

COSMOLOGICAL
CONSTANT

INTO BLACK HOLES

$$ds^2 = f^2(r)\,dt^2 + f^{-2}dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)$$

$$F_{\mu\nu\lambda\rho} = (dA)_{\mu\nu\lambda\rho} = E\sqrt{g}\,\epsilon_{\mu\nu\lambda\rho}$$

Membrane

$$r = R(\tau) \qquad\qquad t_{\pm} = T_{\pm}(\tau)$$

$$ds^2 = d\tau^2 + R^2(\tau)\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)$$

$$1 = f_{\pm}^2\,\dot{T}_{\pm}^2 + f_{\pm}^{-2}\,\dot{R}^2 \quad , \; f = f(R(\tau))$$

$$f_{\pm}^2 = 1 - \frac{2M_{\pm}}{r} - \frac{r^4}{l_{\pm}^2}$$

$$E_{\pm}^2 = \frac{1}{4\pi}\left(\frac{3}{l_{\pm}^2} - \lambda\right)$$

exterior
i[redacted] = initial = +
[redacted]
interior = final = −

$$f_-^2\,\dot{T}_- - f_+^2\,\dot{T}_+ = \mu R$$ First integral of membrane eq. ("Conservation of Energy")

$$E_+ - E_- = q$$ Gauss law

Orientation conventions
- t increases anticlockwise around cosmological horizon
- τ increases when interior is on right side

interior: "−" | exterior "+"

$$\Delta M = M_- - M_+$$

$$\Delta M = \tfrac{1}{2}(\alpha^2 - \mu^2) R^3 - \mu f_+^2 \dot{T}_+ R^2$$

$$= \tfrac{1}{2}(\alpha^2 + \mu^2) R^3 - \mu f_-^2 \dot{T}_- R^2$$

$$\alpha^2 = \frac{1}{l_+^2} - \frac{1}{l_-^2}$$

("Λ jump")
$\Lambda = 3/\ell^2$

Turning points $\qquad \dot{R} = 0$

$$\Delta M = \tfrac{1}{2}(\alpha^2 - \mu^2) R^3 - \varepsilon_+ \mu f_+ R^2$$

$$= \tfrac{1}{2}(\alpha^2 + \mu^2) R^3 - \varepsilon_- \mu f_- R^2 \qquad (*)$$

$$\varepsilon_\pm = \operatorname{sgn} \dot{T}_\pm$$

Mass Diagram

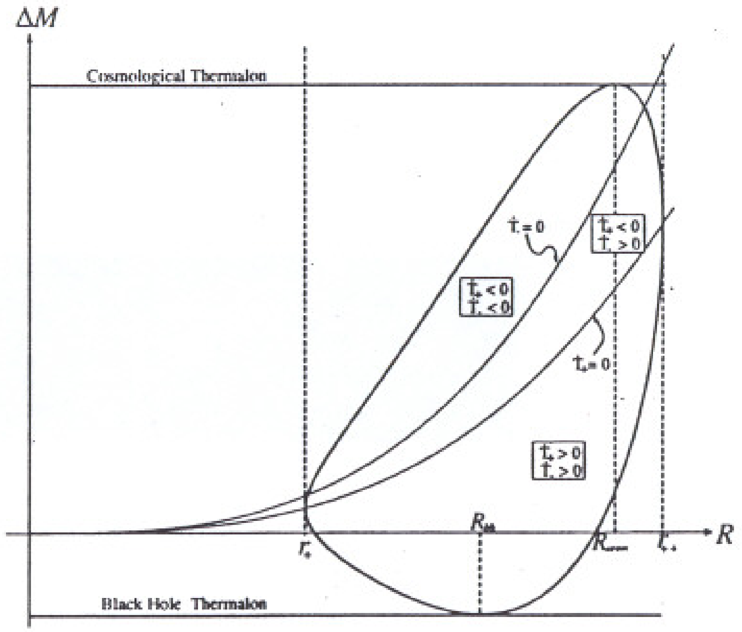


FIG. 2: The closed curve shows the points R where $\dot{R} = 0$ for a given mass gap ΔM. The thermalons are located at the maximum (cosmological thermalon) and minimum (black hole thermalon) of the curve. The curves where R is such that for a given ΔM, $\dot{T}_{\pm} = 0$ are also shown. All the "+" parameters are held fixed.

Decay of de Sitter space through cosmological thermalon

$M_+ = 0$ in mass diagram

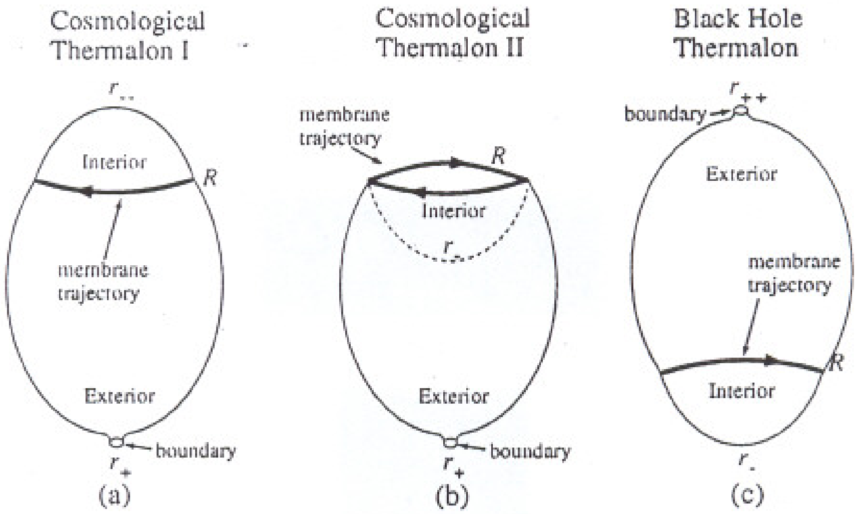
$$M_- = \frac{\mu \ell^2}{3x} \left(1+x^2\right)^{-1/2}$$

$$R^2 = \frac{\ell_+^{\,2}}{1+x^2}$$

$$x = \frac{3}{4}\left[-\gamma + \left(\gamma^2 + \frac{8}{9}\right)^{1/2}\right]$$

$$\gamma = \frac{\ell_+ (\alpha^2 - \mu^2)}{2\mu}$$

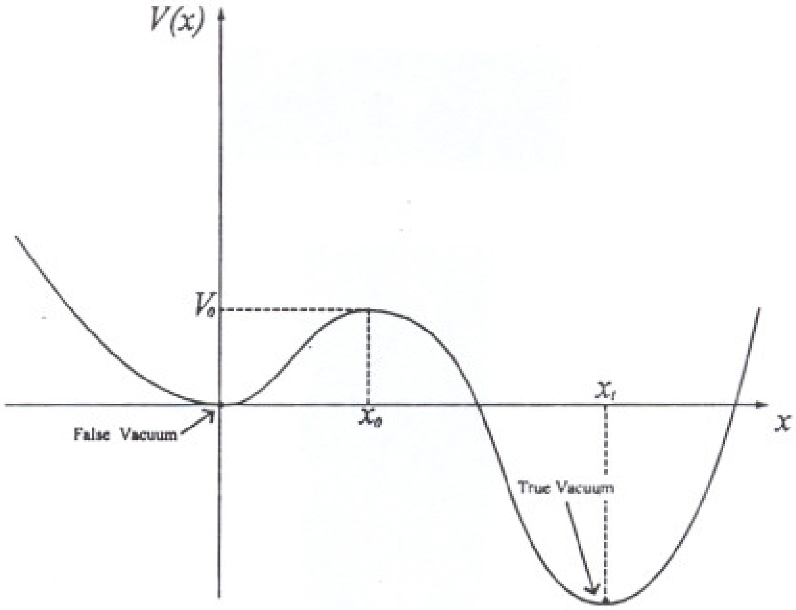
Can also compute Nariai threshold ℓ_N



Transition between (a) and (b)
occurs when $r_- = r_{--}$: Nariai threshold
smooth $M_- \le \frac{\ell_-}{3\sqrt{3}}$ maintained
throughout. Action continuous

Note however

above threshold (a) r increases from r_+ to r_{--}
(b) r increases from r_+ to R
and decreases to minimum
value at r_-



V(x)
V0
x0
x1
x
False Vacuum
True Vacuum

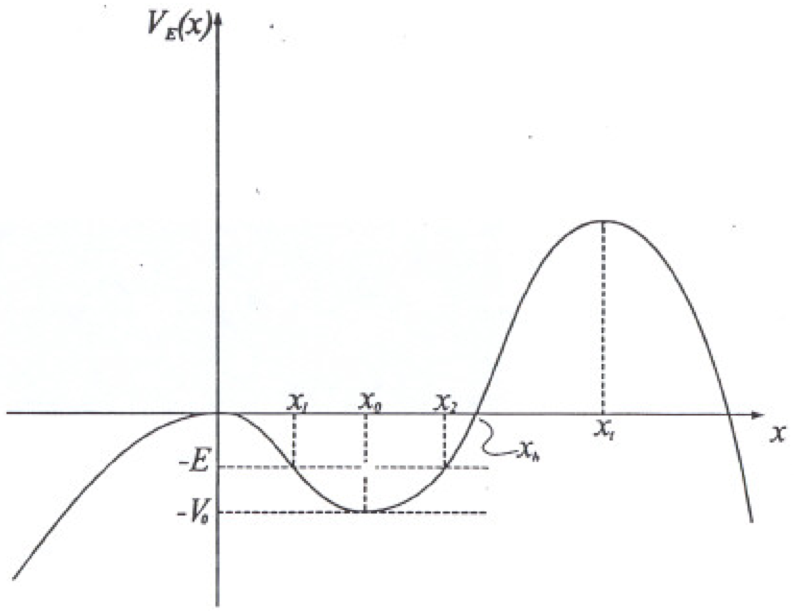


FIG. 2: The Euclidean system consist in the particle placed in a potential $V_E = -V$. The Euclidean action evaluated in particular orbits dominate the path integral describing particular decay processes. At zero temperature, is the instanton which starts at rest at $x = 0$, $t = -\infty$, bounces at $x = x_b$ and comes back to $x = 0$ at $t = \infty$. At higher inverse temperatures β, the corresponding instantons are periodic orbits, bounded between two turning points, x_1, x_2, with period $\beta\hbar$. This happens at some particular value of the energy $-E$. When the temperature is high enough the static orbit at the bottom of the potential well $x = x_0$, the thermalon, becomes dominant.

Probability

$$\Gamma = A e^{-B/\hbar} \left(1 + O(\hbar)\right)$$

$$-B = I_{\text{thermalon}}$$

Action includes contribution $\frac{1}{4}$ area of horizon which is not boundary

$$I_{\text{thermalon}} = \frac{1}{4}\left[A(r_{--}) - A(r_{++})\right] - \frac{\mu V_3}{4\pi\xi} + \alpha^2 \frac{3}{8\pi} V_4$$

below threshold

includes back-ground substraction

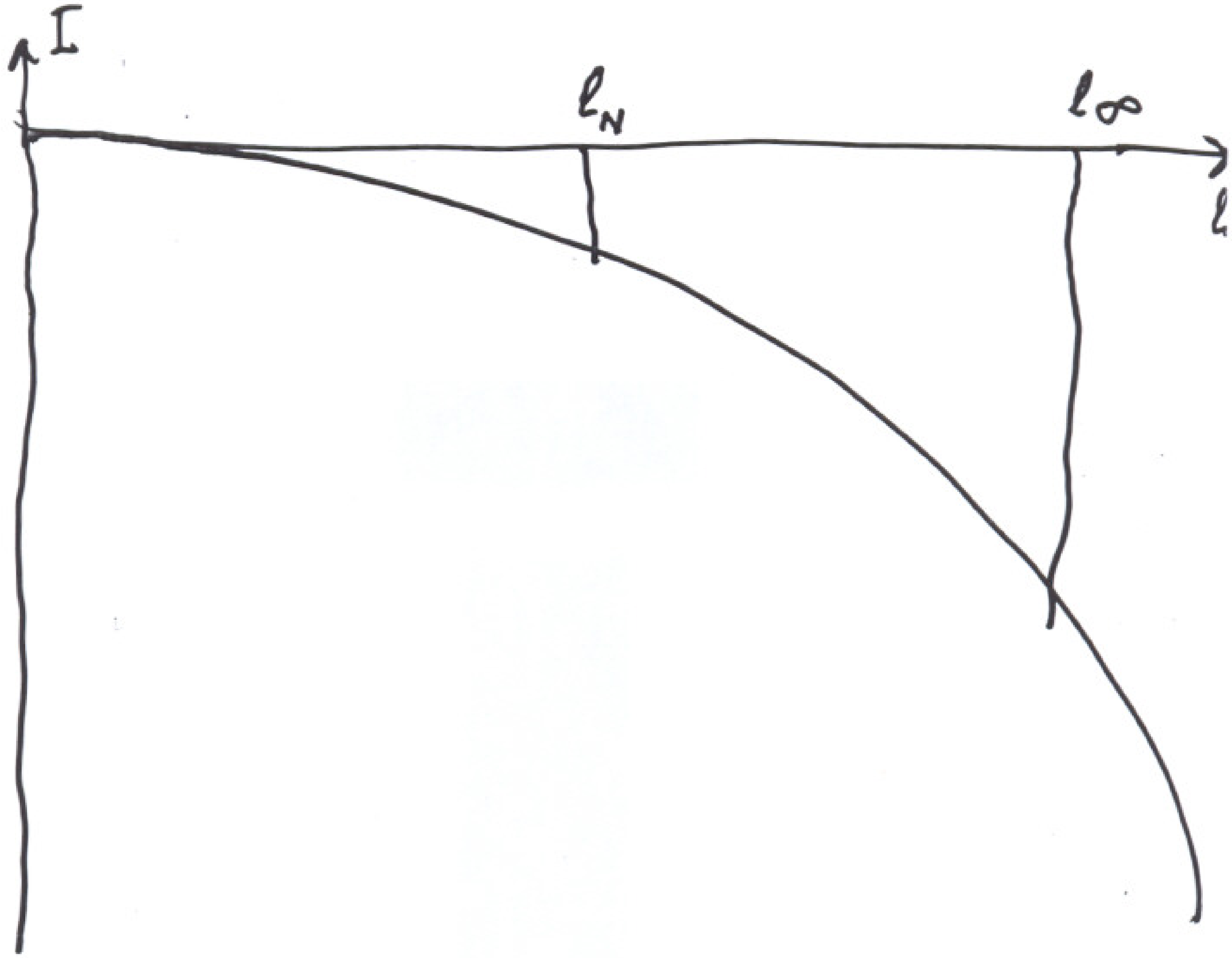
$$I_{\text{thermalon}} = \frac{1}{4}\left[A(r_-) - A(r_{++})\right]$$

$$-\frac{\mu}{4\pi} V_3 + \alpha^2 \frac{3}{8\pi} V_4$$

above threshold

TABLE I: Instantons and Thermalons Compared and Contrasted.

Solution of the classical Equation of motion	Process that the solution describes when used to dominate path integral	Dependence on Euclidean time for motion in one-dimensional potential	Range of validity if used to approximate decay rate by steepest descent	Formula for decay rate
Instanton at zero temperature	Tunneling through potential barrier	Time dependent. Starts at x_1 at $t = -\infty$, bounces at x_2 and comes back to x_1 at $t = \infty$. The bounce is localized in time (for large values of $\|t\| > T$, $x \ll (x_2 - x_1)$).	Dominates for $$\beta \sim \infty$$	$$\Gamma = \frac{2}{\hbar}\mathrm{Im}F = \frac{2}{\hbar}\mathrm{Im}E$$
Instanton at non-zero temperature	Tunneling through potential barrier at non-zero temperature	Time dependent, starts at x_1, bounces at x_2 and comes back to x_1 after a full period $\Delta t = \beta$	Dominates for $$\frac{2\pi}{\omega} \ll \beta\hbar < \infty$$	$$\Gamma = \frac{2}{\hbar}\mathrm{Im}F$$
Thermalon	Going over the barrier by a thermal kick	Time independent. Stands at the bottom of the Euclidean potential for all times	Dominates for $$0 < \beta\hbar \ll \frac{2\pi}{\omega}$$	$$\Gamma = \frac{\omega\beta}{\pi}\mathrm{Im}F$$
			In overlap region, $\beta\hbar \sim \frac{2\pi}{\omega}$, both solutions should in general be included. Here ω is the frequency of oscillations in Euclidean time of perturbations around the thermalon. It is assumed that there is only one such stable Euclidean mode. For motion in a potential $\omega = V''(x_T)$	F has imaginary part because extremum is a saddle point due to unstable state



$$\frac{1}{l_\infty^2} = \frac{4}{3} q \left(\pi q + \sqrt{-\pi \lambda} \right)$$

above l_∞ cosmological constant is negative
transition to anti de Sitter.

What about actual universe

Planck units $\quad \Lambda_{now} \sim 10^{-120}$

Age of universe $\quad 10^{60}$

use as time available for estimates, although perhaps Λ relaxed only during part of history of universe.

If ℓ_+ started at Planck scale

$\ell_+ = 1$ initially, $\quad \ell_+ = 10^{60}$ now

each nucleation reduces ℓ_+^{-2} by $\alpha^2 \sim \Lambda_{now}$ ~~...~~

$\sim 10^{-120}$

So need 10^{120} events to occur in 10^{60} planck time units

$$\Gamma = \frac{10^{120}}{10^{60}\,\ell_+^3} = \frac{10^{60}}{\ell_+^3} \qquad (*)$$

location of center

For to be realizable in available range ~~...~~ $1 < \ell_+ < 10^{60}$ need

$$10^{-120} < Ae^{-B} < 10^{60}$$

$\alpha^2 \sim 10^{-120}$

B not too small for semiclassical regime

need detailed analysis of A

$\left(A = \frac{\omega\beta}{\pi} \times \dots \right.$ involves membrane deformation determinant, ...)