GIANT GRAVITONS, FUZZY CYLINDERS AND FUZZY CP^2

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GRAVITONS IN $AdS_m \times S^n$ AND THE STRINGY EXCLUSION PRINCIPLE

- In the AdS/CFT correspondence:

<u>CFT</u>	<u>AdS</u>
Family of chiral primary operators	Graviton states with angular momentum in S^n
Maximum weight (finite rank of the gauge group)	Upper bound on the angular momentum $\Rightarrow$ Stringy exclusion principle
	(Maldacena & Strominger '98)

- Mc. Greevy, Susskind & Toumbas '00: The gravitons expand into the spherical part of the geometry (giant gravitons) with

  radius $\sim$ angular momentum

  Radius $<$ radius of $S^n \Rightarrow$ Upper bound on the angular momentum

- The solution:

- Full line element for the metric on $AdS_m \times S^n$:

$$ds^2_{AdS} = -(1+\frac{r^2}{L^2})dt^2 + (1+\frac{r^2}{L^2})^{-1}dr^2 + r^2 d\Omega^2_{m-2}$$

$$ds^2_{sph} = L^2(d\theta^2 + \cos^2\theta d\phi^2 + \sin^2\theta d\Omega^2_{n-2})$$

- (n-1)-form potential on S^n:

$$C^{(n-1)}_{\phi\chi_1\cdots\chi_{n-2}} = L^{n-1}\sin^{n-1}\theta\sqrt{g_\chi}$$

- Find stable test brane solutions where an (n-2)-brane has expanded on the S^n to a sphere with fixed θ while orbiting the S^n in the ϕ direction:

$$\theta = \text{const}\,, \qquad \phi = \phi(\tau)\,, \qquad r = 0$$

$$\Rightarrow ds^2 = -dt^2 + L^2\cos^2\theta d\phi^2 + L^2\sin^2\theta d\Omega^2_{n-2}$$

$L^2\sin^2\theta d\Omega^2_{n-2}$ ↓ spherical (n-2)-brane with radius $= L\sin\theta$

$L^2\cos^2\theta d\phi^2$ ↓ moving in S^n

$$S_{(n-2)} = -T_{n-2} \int \sqrt{\det g} + T_{n-2} \int C^{(n-1)} =$$

$$= \frac{\tilde{N}}{L} \int d\tau [- \sin^{n-2}\theta \sqrt{1 - L^2 \cos^2\theta \dot{\phi}^2} + L \sin^{n-1}\theta \dot{\phi}]$$

where

$$\int d\Omega_{n-2} = A_{n-2} \qquad \text{and} \qquad A_{n-2} T_{n-2} = \frac{\tilde{N}}{L^{n-1}}$$

ϕ cyclic $\Rightarrow$ P_ϕ conserved:

$$H = \frac{P_\phi}{L} \sqrt{1 + \tan^2\theta (1 - \frac{\tilde{N}}{P_\phi} \sin^{n-3}\theta)^2} = H(\theta)$$

$\Rightarrow$ Equilibrium radius of the spherical brane determined by: $H'(\theta) = 0$

$$\Leftrightarrow \begin{cases} \theta = 0 \rightarrow H = P_\phi / L : & \text{point-like graviton} \\ \sin^{n-3}\theta = P_\phi / \tilde{N} \rightarrow H = P_\phi / L : & \text{giant graviton} \end{cases}$$

For the giant graviton:

$$R = L \sin\theta \leq L \Leftrightarrow \sin\theta \leq 1 \Rightarrow$$

$\Rightarrow P_\phi \leq \tilde{N} \Rightarrow$ Upper bound for the angular momentum

- $M = P_\phi/L\,; \qquad Q = P_\phi/L \qquad \Rightarrow$ BPS
- $\dot{\phi} = 1/L$ as point-like graviton
- Preserves same susys than point-like graviton:

$$(1 + \Gamma^{0\phi})\epsilon = 0$$

SUSY algebra:

$$\{Q_\alpha, Q_\beta\} = P_0 + \Gamma^{012} Z_{12} \qquad \text{(membrane)}$$

$$\{Q_\alpha, Q_\beta\} = P_0 + \Gamma^{0i} P_i \qquad \text{(grav.wave)}$$

(Bergshoeff, Kallosh & Ortín '93)

$P^0 = P_i \Rightarrow \{Q_\alpha, Q_\beta\} = P_0(1 + \Gamma^{0i})$

P_i: electric charge of the graviton

But, Grisaru, Myers & Tafjord '00; Hashimoto, Hirano & Itzhaki '00: The gravitons can also expand into a sphere in the AdS part (dual giant gravitons) with

radius $\sim$ angular momentum

AdS non-compact $\Rightarrow$ No upper bound on the angular momentum

Realisation of the stringy exclusion principle?

Distinct CFT states or different descriptions of the same state?

Quantum mechanical mixing?

(Grisaru, Myers & Tafjord '00; Hashimoto, Hirano & Itzhaki '00; Das, Jevicki & Mathur '00; Balasubramanian et al '01; Corley, Jevicki & Ramgoolam '01; Myers & Tafjord '01; Bena & Smith '04; Berenstein '04;..)

Giant graviton: $(n-2)$-brane with S^{n-2} topology and angular momentum P_ϕ. "Macroscopical" $(R >> l_s)$

Can we give a "microscopical" description, in terms of blown up gravitons ??

Connect with known examples of expanded brane configurations: Myers dielectric effect

→ Giant gravitons as dielectric gravitational waves:

- B. Janssen, Y.L. & D. Rodríguez-Gómez, hep-th/0406148
- B.Janssen & Y.L., hep-th/0207199, NPB
- B. Janssen, Y.L. & D. Rodríguez-Gómez, hep-th/0303183, NPB

Example: N D0's in a constant RR $F^{(4)}$:

$$[X^i, X^j]C^{(3)}_{ijk}$$

$$\int d\tau \mathrm{STr}(i_X i_X C^{(3)}) = -\frac{1}{3}\int d\tau \mathrm{STr}(X^j X^i X^k)F^{(4)}_{0ijk} + \ldots$$

$F^{(4)}_{0ijk} = 2f\epsilon_{ijk} \rightarrow$ Ground state: N D0's expanded into a fuzzy S^2:

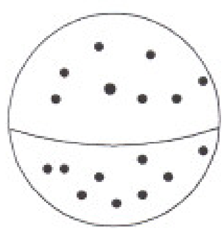
$$f = \frac{R}{\sqrt{N^2-1}}$$

$$X^i = \frac{f}{2}J^i, \qquad [J^i, J^j] = 2i\epsilon^{ijk}J^k$$

$$\Rightarrow \sum_{i=1}^{3}(X^i)^2 = (\frac{f}{2})^2(N^2-1)\mathbb{1} = R^2\mathbb{1}$$

$C^{(3)}$ dipole moment $\Rightarrow$ Fuzzy D2-brane

Regime of validity: $R << \sqrt{N}l_s$



For infinite number of D0's: The N D0's expanding into a fuzzy D2 can also be described as a D2 with S^2 topology and D0-brane charge (Emparan '97)

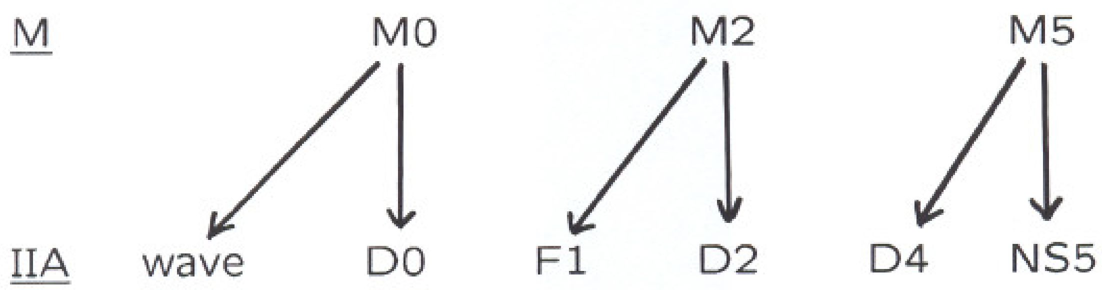
Regime of validity: $R >> l_s$

GIANT GRAVITONS IN $AdS_m \times S^n$ MICROSCOPICALLY

- Action for coincident gravitons
- Dielectric couplings

The action for coincident gravitons

D0-branes are gravitons in M-theory, moving along the 11th direction:



$\Rightarrow$ Uplift the action for N D0-branes!

Also, Matrix Theory calculation:

Matrix string theory: (Non-Abelian) Type IIA strings with non-zero light-cone momentum

Sen-Seiberg limit + Static gauge $\longrightarrow$ (Non-Abelian) massless particles with spatial momentum

$\downarrow$

IIA gravitational waves

(Janssen & Y.L. '02)

Dielectric couplings?: Matrix string theory in a weakly curved background (Schiappa '00; Brecher, Janssen & Y.L. '01):

$$S = S_{\text{flat}} + S_{\text{linear}}$$

$\supset$ dielectric couplings

$$S_{\text{linear}} = \int \text{STr}\{\frac{1}{2}h_{AB}T^{AB} + C^{(3)}_{ABC}J^{ABC} + C^{(6)}_{ABCDEF}M^{ABCDEF}\}$$

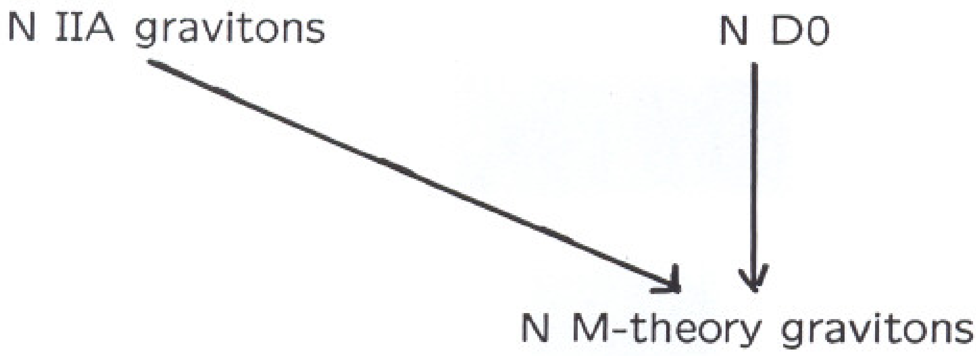
(Kabat & Taylor '98)

$\Big\downarrow$ Sen-Seiberg limit

$$S^{(\text{linear})}_{ND0} = \int \text{STr}\{\frac{1}{2}h_{ab}I^{ab}_h + \phi I_\phi + B^{(2)}_{ab}I^{ab}_s + C^{(1)}_a I^a_0 + C^{(3)}_{abc}I^{abc}_2 + \ldots\}$$

Precise agreement with the linear expansion of Myers action:

$$C^{(1)}_a I^a_0 = P[C^{(1)}]\,; \qquad C^{(3)}_{abc}I^{abc}_2 = iP[i_X i_X C^{(3)}]$$



→ Giant gravitons in $AdS_7 \times S^4$ and $AdS_4 \times S^7$. For fuzzy S^2: Janssen & Y.L. '02. Perfect agreement with the macroscopical description!

→ IIB gravitons (T-duality) ⇒ $AdS_5 \times S^5$: Fuzzy S^3: S^1 bundle over a fuzzy S^2. (Janssen, Y.L. & Rodríguez-Gómez '03).

$AdS_3 \times S^3 \times T^4$: Fuzzy cylinder. (Janssen, Y.L. & Rodríguez-Gómez '04).

Perfect agreement with the macroscopical description!

For N M-theory gravitons:

$$S = T_0 \int d\tau \mathrm{STr}\{-k^{-1}\sqrt{E_{00} + E_{0i}(Q^{-1} - \mathbb{1})^i_k E^{kj} E_{j0}}\sqrt{\det Q}$$

$\frac{g_{i\phi}}{g_{\phi\phi}} \leftrightarrow B^{(2)}_{i\phi}$

$$-k^{-2}k_i\partial X^i + i(i_X i_X)C^{(3)} + \frac{1}{2}(i_X i_X)^2 i_k C^{(6)} + \ldots\}$$

k^μ: Killing vector pointing along the direction of propagation (isometric $\leftrightarrow$ momentum eigenstate ($\leftrightarrow$ superstar solutions smeared in the ϕ-direction (Myers & Tafjord '01)))

$$E = \mathcal{G} + k^{-1}(i_k C^{(3)})\,, \qquad Q^i_j = \delta^i_j + ik[X^i, X^k]E_{kj}$$

In the Abelian limit: Legendre transformation $\rightarrow$

$$S[\gamma] = -\frac{T_0}{2}\int d\tau\sqrt{|\gamma|}\gamma^{-1}\partial X^\mu \partial X^\nu g_{\mu\nu}$$

EXAMPLE: THE GIANT GRAVITON IN $AdS_7 \times S^4$

Macroscopically: Spherical M2-brane with $R = L\sin\theta$ and $P_\phi \neq 0$.

- Fuzzy ansatz:

$$X^i = \frac{L\sin\theta}{\sqrt{N^2-1}} J^i \,, i = 1,2,3$$

- The gravitons have charge $P_\phi \Rightarrow$ Identify ϕ as the isometric direction

- $C^{(3)}_{\phi ij} = -\epsilon_{ijk} X^k$

Microscopical potential:

$$V(X) = \frac{T_0}{L\cos\theta} \mathrm{STr}$$

$$\{\sqrt{\mathbb{1} - \frac{4L\sin\theta}{\sqrt{N^2-1}} X^2 + \frac{4L^4\sin^2\theta\cos^2\theta}{N^2-1} X^2 + \frac{4L^2\sin^2\theta}{N^2-1}(X^2)^2}\}$$

When $L\sin\theta << \sqrt{N^2-1}$:

$$V(\theta) = \frac{NT_0}{L}\sqrt{1 + \tan^2\theta(1 - \frac{2L^3}{\sqrt{N^2-1}}\sin\theta)^2}$$

$$NT_0 = P_\phi \,, \qquad \frac{2L^3}{N} = \frac{\tilde{N}}{P_\phi}$$

$\Rightarrow$ Exact agreement!

GIANT GRAVITONS IN $AdS_3 \times S^3 \times T^4$

IIB background $\Rightarrow$ Action for Type IIB gravitons

T-duality direction isometric

$$E = \mathcal{G} - k^{-1} l^{-1} e^{\phi} (i_k i_l C^{(4)})$$

Macroscopical description: Bound state of D1- and D5-branes (wrapped on T^4) wrapped on S^1 ($\subset\ S^3$)

Microscopically: Embed the circle in a fuzzy cylinder ($\leftrightarrow$ supertube):

$$[X^1, X^2] = 0\,, \qquad [X^1, X^3] = i f X^2\,, \qquad [X^2, X^3] = -i f X^1$$

Quadratic Casimir: $(X^1)^2 + (X^2)^2 = R^2 \mathbb{1} \Rightarrow$ Fuzzy circle

X^3? $\leftrightarrow$ World-volume scalar in the action, with no geometrical interpretation:

$$Z \to \omega \qquad \text{such that} \qquad \mathcal{F} = \partial\omega + i_l B^{(2)}$$

- $C^{(2)}_{\phi i} \neq 0$ couples in

$$Q^i_j = \ldots + [X^i, \omega](i_k C^{(2)})_j$$

- Irreducible representations with infinite dimension $\Rightarrow$ Infinite number of gravitons

Microscopical potential (exact):

$$V(\theta) = \frac{\mathrm{Tr}\mathbb{1} T_0}{L}\sqrt{1 + \tan^2\theta(1 - f(nQ_5 + mQ_1))^2}$$

Minimum energy condition: $f = (nQ_5 + mQ_1)^{-1}$.But, giant graviton with arbitrary size.

Macroscopical Hamiltonian:

$$H = \frac{P_\phi}{L}\sqrt{1 + \tan^2\theta(1 - \frac{2\pi T_1(nQ_5 + mQ_1)}{P_\phi})^2}$$

Minimum energy condition: $P_\phi = 2\pi T_1(nQ_5 + mQ_1)$. Giant graviton with arbitrary size.

- $\mathrm{Tr}\mathbb{1} T_0/l = P_\phi = T_0/f \Rightarrow$ Exact agreement!

$$(\omega_{mn} = f(m - 1/2)\delta_{m,n} \Rightarrow l = f\mathrm{Tr}\mathbb{1})$$

- Non-commutative cylinder ($f \neq 0$) $\leftrightarrow$ ω has no geometrical meaning $\rightarrow S^1$
- Becomes commutative in the classical limit $Q_1, Q_5 \rightarrow \infty$.

Unrelated to the stringy exclusion principle $\leftrightarrow$ Mixed giant gravitons expanding at the same time in AdS_3 and in S^3

Giant gravitons are not the SUGRA duals of the chiral primary states of the D1-D5 CFT (Lunin, Mathur & Saxena '02..)

GIANT GRAVITONS IN $AdS_4 \times S^7$

$S^5_{\text{fuzzy}} \leftrightarrow$ Find $SO(6)$- covariant matrix realisations of the condition $\sum_{i=1}^{6}(X^i)^2 = \mathbb{1}$:

$\rightarrow$ Complicated technically (Ramgoolam '01, '02)

S^5_{fuzzy} as an S^1-bundle over CP^2_{fuzzy}

CP^2_{fuzzy}: Trivedi & Vaidya '00:

CP^2: $SU(3)/U(2)$	S^2: $SU(2)/U(1)$
Embed CP^2 in flat space: $\mathbb{R}^8$	$S^2 \subset \mathbb{R}^3$
$X^i = fT^i\,; i = 1 \ldots 8$	$X^i = fT^i\,; i = 1, 2, 3$

T^i: Generators of $SU(3)$ in the irrep $(m, 0)$ ($\leftrightarrow$ isotropy group $U(2)$)

$\rightarrow$ Non-commutative generalisation of $SU(3)/U(2)$

Checking agreement with macroscopical description...

CONCLUSIONS

- Action for coincident M-theory and Type II gravitons suitable for the study of $AdS_m \times S^n$ backgrounds
- With this action: Giant gravitons as dielectric gravitational waves
- In perfect agreement with the macroscopical description