The BFKL Pomeron in AdS/CFT

Lorenzo Cornalba
University of Milano Bicocca & Centro Fermi

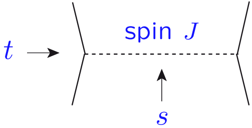
The IST String Fest – Lisbon, June 2009

Introduction I

- High energy phenomena probe relevant features of interactions
- In flat space string theory they are controlled by the leading Regge trajectory of states
- In perturbative YM theory (QCD and susy completions) dominated by Pomeron exchange
- High energy strings in holographic backgrounds will interpolate between these two regimes
  - We will present a general formalism valid both at weak and at strong coupling
  - At large 't Hooft coupling, analyze gravi–Reggeon exchange in AdS and recover flat space interactions
  - At weak coupling, use holography to correctly analyze known pQCD results

High Energy Scattering in Flat Space I

- Scattering in flat space in the Regge limit $s \gg -t$
- Single spin J exhange in the T-channel



$$\sim \quad s^{J}$$

Maximal spin dominates

- J unbounded $\rightarrow$ High s behavior controlled by leading Regge pole in the complex J plane

$$\mathcal{A}_{\text{tree}}(s,t) \sim \beta(t) \cdot s^{j(t)}$$

- Strings in flat space

$$j(t) = 2 + \frac{\alpha'}{2} t$$

- Violation of unitarity at ultra-high energies

High Energy Scattering in Flat Space II

- Partial wave decomposition in the S-channel

$$\mathcal{A} \sim \sum_{J \geq 0} C_J \left(\frac{t}{s}\right) e^{2i\delta_J(s)} \qquad \qquad (\text{Im}\delta_J(s) \geq 0)$$

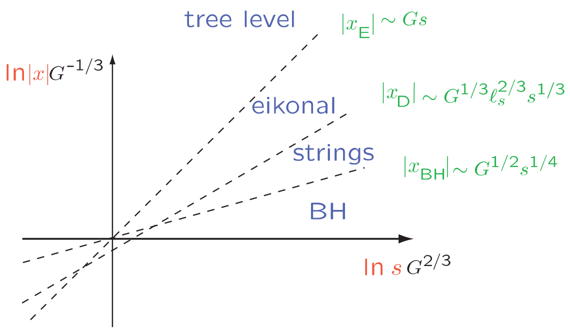
- In the Regge limit

$$\mathcal{A} \sim 2s \int d^3x\, e^{iq\cdot x + 2i\delta(s,x)} \qquad \qquad \begin{pmatrix} q^2 = -t \\ |x| = 2J/\sqrt{s} \end{pmatrix}$$

- Eikonal resummation of tree interaction

$$\mathcal{A}_{\text{tree}} \sim 4is \int d^3x\, e^{iq\cdot x}\, \delta(s,x)$$

High Energy Scattering in Flat Space III



- Eikonal limit when phase shift $\mathcal{A}_{\text{tree}}(s,x)/s \sim Gs/|x| \sim 1$. Multi gravi–reggeon exchange of linear gravitons

- In AdS_5 we expect similar physics with
  - Extra scale of the AdS radius ℓ
  - Transverse space H_3 instead of $\mathbb{E}^3$

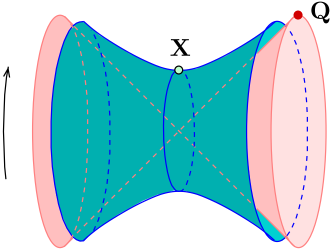
High Energy Interactions in AdS I

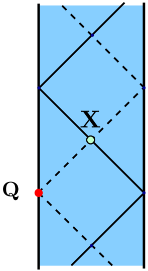
- AdS$_5$ given by

$$\mathbf{X}^2 = -\ell^2 \qquad (\ell = 1)$$
$$\mathbf{X} \in \mathbb{M}^2 \times \mathbb{M}^4$$

Boundary points given by rays

$$\mathbf{Q}^2 = 0 \qquad \mathbf{Q} \sim \lambda \mathbf{Q}$$





High Energy Interactions in AdS II

- Boundary points behave as momenta for AdS interactions. External wave functions

$$\frac{1}{(-2\mathbf{X}\cdot\mathbf{Q})^{\Delta}} \sim \int d\lambda\, e^{i\mathbf{X}\cdot\lambda\mathbf{Q}}\, \lambda^{\Delta-1}$$

  dual to CFT operator of dimension Δ

- Scalar amplitudes

$$A(\mathbf{Q}_i)$$

  depend on invariants

$$\mathbf{Q}_{ij} = -(\mathbf{Q}_i + \mathbf{Q}_j)^2$$

  and are homogeneous

$$A(\cdots, \lambda\mathbf{Q}_i, \cdots) = \lambda^{-\Delta_i} A(\cdots, \mathbf{Q}_i, \cdots)$$

High Energy Interactions in AdS III

- Consider CFT correlator

$$\langle \mathcal{O}_1(\mathbf{Q}_1)\,\mathcal{O}_1^\star(\mathbf{Q}_3)\,\mathcal{O}_2(\mathbf{Q}_2)\,\mathcal{O}_2^\star(\mathbf{Q}_4)\rangle = \frac{1}{\mathbf{Q}_{13}^{\Delta_1}\mathbf{Q}_{24}^{\Delta_2}}\mathcal{A}\,(\text{cross ratios})$$

- Cross ratios

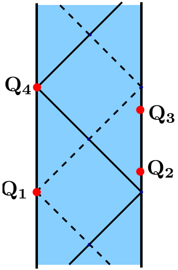
$$\frac{\mathbf{Q}_{13}\mathbf{Q}_{24}}{\mathbf{Q}_{12}\mathbf{Q}_{34}} \qquad\qquad \frac{\mathbf{Q}_{14}\mathbf{Q}_{23}}{\mathbf{Q}_{12}\mathbf{Q}_{34}}$$

- Regge kinematics

$$\mathbf{Q}_{13},\,\mathbf{Q}_{24} \to 0$$

- Basic example in $\mathcal{N}=4$ SYM

$$\mathcal{O}_1 = \text{Tr}(Z^2) \qquad \mathcal{O}_2 = \text{Tr}(W^2)$$
$$\mathcal{A} = 1 + N^{-2}\mathcal{A}_{\text{planar}} + \cdots$$



T Channel View I

- Conventions

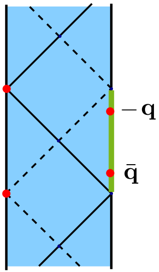
$$\mathbf{X} = (X^+, X^-, x) \in \mathbb{M}^2 \times \mathbb{M}^4$$
$$x = (x^+, x^-, \mathbf{x}) \in \mathbb{M}^4$$

- Scattering kinematics

$$\mathbf{Q}_1 = (0,1,0) \qquad \mathbf{Q}_3 = -(q^2, 1, -q)$$
$$\mathbf{Q}_2 = (1, \bar{q}^2, \bar{q}) \qquad \mathbf{Q}_4 = -(1,0,0)$$

with

$$q, \bar{q} \in \text{Future light cone} \subset \mathbb{M}^4$$



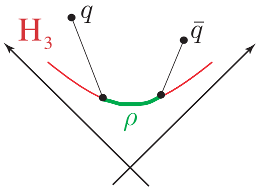
T Channel View II

- Transverse conformal group

$$SO(3,1) \qquad q, \bar{q} \text{ vectors}$$
$$SO(1,1) \qquad q, \bar{q} \to \lambda q, \lambda^{-1}\bar{q}$$

- Cross ratios

$$\sigma^2 = q^2 \bar{q}^2 \qquad \sigma \cosh \rho = -q \cdot \bar{q}$$

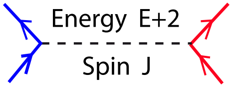


- Regge kinematics

$$\sigma \to 0 \qquad \rho \text{ fixed}$$

T Channel View III

- T–channel exchange $\mathcal{T}_{E,J}(\sigma,\rho)$ of a conformal block of



- Euclidean OPE for $\sigma \rightarrow 0$

$$\mathcal{T}_{E,J} \sim \sigma^{2+E}$$

- Scattering regime

$$\mathcal{T}_{E,J} \;\sim\; \sigma^{1-J}\,\Pi_E(\rho)$$

$\Pi_E(\rho)$ propagator of energy $1+E$ in H_3

- General spin J contribution to correlator

$$\sigma^{1-J}\int d\nu\;\alpha_J(\nu)\;\Omega_{i\nu}(\rho) \qquad\qquad \left(-\Box_{H_3} = 1+\nu^2\right)$$

T Channel View IV

- Maximal spin dominates. If spin J unbounded resum contributions. Leading Regge pole at $J = j(\nu)$ gives

$$\mathcal{A}_{\text{planar}} \sim \int d\nu \, \sigma^{1-j(\nu)} \, \alpha(\nu) \; \Omega_{i\nu}(\rho)$$

- In $\mathcal{N} = 4$ SYM

$$j(\nu, g) \qquad\qquad \left(g^2 = g_{\text{YM}}^2 N\right)$$
$$\alpha(\nu, g)$$

with $j(\nu, g)$ the spin of the twist two operator of dimension $2 + i\nu$

S Channel Eixonal Resummation I

- Impact parameter representation

$$\mathcal{A} \sim |q\bar{q}|^4 \int_{\mathrm{M}} dx d\bar{x}\ e^{-iq\cdot x - i\bar{q}\cdot\bar{x}}\ e^{2i\Delta(x,\bar{x})}$$

- Cross ratios

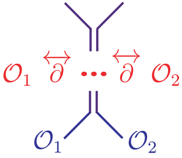
$$S = |x|\,|\bar{x}|$$

B impact parameter on H_3

- No interaction

$$\mathcal{A} = 1 \qquad\qquad \Delta = 0$$

- Lattice sum



$$1 = \sum_{E,J} \mathcal{S}_{E,J}$$

$$E = \Delta_1 + \Delta_2 + J + 2n$$

S Channel Eixonal Resummation II

- For large E, J replace $\sum$ with $\int_{\mathsf{M}} dx d\bar{x}$ where

$$E = \sqrt{S} \cosh \left(B/2 \right) \qquad\qquad J = \sqrt{S} \sinh \left(B/2 \right)$$

- Phase shift determined eikonally by planar amplitude

$$\Delta \sim \frac{1}{N^2} \int d\nu \; S^{j(\nu)-1} \beta \left(\nu \right) \; \Omega_{i\nu} \left(B \right)$$

with

$$\alpha \left(\nu \right) = V_{\text{min}} \left(\nu, j \left(\nu \right) \right) \cdot \beta \left(\nu \right) \cdot \bar{V}_{\text{min}} \left(\nu, j \left(\nu \right) \right)$$

- High energy unitarity

$$\text{Im}\, \Delta \left(S, B \right) \geq 0$$

- Anomalous dimension $-2\Delta/\pi$ of double trace $\mathcal{O}_1 \partial \cdots \partial \mathcal{O}_2$ of dimension E and spin J for elastic AdS interactions

Momentum Space I

- Correlator in momentum space

$$\langle \mathcal{O}_1(p_1)\,\mathcal{O}_1^\star(p_3)\,\mathcal{O}_2(p_2)\,\mathcal{O}_2^\star(p_4)\rangle$$

with

$$t\ ,\ \ p_i^2 \quad \text{fixed} \qquad\qquad s\to\infty$$

- AdS Poincarè coordinates

$$x^+\ ,\ x^-\ ,\underbrace{\mathbf{x}\ ,\ r}_{\mathrm{H}_3}$$

- External states

$$e^{-i\sqrt{s}x^-}\ e^{i\mathbf{p}_1\cdot\mathbf{x}}\,f_1(r) \qquad\qquad \left(r^2\sim 1/p_1^2\right)$$

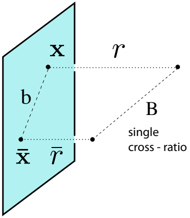
- Amplitude

$$s\int\frac{dr d^2\mathbf{x}}{r^3}\,f_1(r)\,f_3(r)\,e^{i\mathbf{p}\cdot\mathbf{x}}\cdot\int\frac{d\bar{r}d^2\bar{\mathbf{x}}}{\bar{r}^3}\,f_2(\bar{r})\,f_4(\bar{r})\,e^{-i\mathbf{p}\cdot\bar{\mathbf{x}}}\cdot e^{2i\Delta(S,B)}$$

Momentum Space II

- Phase shift Δ depends on

$$S = sr\bar{r}$$
$$\cosh B = \frac{1}{2r\bar{r}}\left[r^2 + \bar{r}^2 + (\mathbf{x} - \bar{\mathbf{x}})^2\right]$$



- Final answer

$$s\int d^2\mathbf{b}\, e^{i\mathbf{p}\cdot\mathbf{b}}\, e^{2i\delta(s,\mathbf{b})}$$

with

$$e^{2i\delta(s,\mathbf{b})} = s\int \frac{dr}{r^3} f_1(r) f_3(r) \cdot \int \frac{d\bar{r}}{\bar{r}^3} f_2(\bar{r}) f_4(\bar{r}) \cdot e^{2i\Delta(S,B)}$$

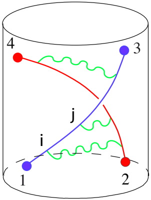
Large 't Hooft Coupling I

- Gravitational interaction in AdS_5 with

$$G \sim \frac{\ell^3}{N^2}$$

- Along $\mathcal{O}_1$ trajectory high energy states follow null geodesics in AdS parameterized by affine parameter x^+ and labelled by $x^-$and a point $\mathbf{x}$, r in H_3 with propagator

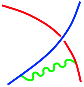
$$\frac{1}{\sqrt{s}}\,\theta(x_j^+ - x_i^+)\cdot$$
$$\delta(x_j^- - x_i^-)\,\delta_{H_3}(\mathbf{x}_j, r_j|\mathbf{x}_i, r_i)$$



Large 't Hooft Coupling II

- For graviton exchange phase given by tree level interaction between geodesics

$$\Delta(S,B) = i\int dx^+ d\bar{x}^-$$



$$\simeq \frac{G}{\ell^3}\, S\, \Pi_2(B)$$

with $\Pi_2\left(B\right)$ propagator in H_3 of scalar dimension 3

- For $g^2 \to \infty$

$$\begin{aligned} j\left(\nu, g\right) &\sim 2 \\ \beta\left(\nu, g\right) &\simeq \frac{\pi^2}{4+\nu^2} \end{aligned}$$

- Anomalous dimension of double trace $\mathcal{O}_1 \partial \cdots \partial \mathcal{O}_2$ of large dimension E and spin J

$$\frac{1}{4N^2}\frac{(E-J)^4}{EJ} \qquad \left(\text{Exact in } 1/N^2\right)$$

Including String Effects I

- Regge trajectory of the graviton $j(\nu, g)$ with

$$g^2 = \frac{\ell^4}{\alpha'^2} = g_{\mathrm{YM}}^2 N$$

- Flat space limit

$$\lim_{\ell \to \infty} j\left(\ell\sqrt{-t}, \ell^2/\alpha'\right) = 2 + \frac{\alpha'}{2} t$$

- Energy momentum tensor $j(\pm 2i, g) = 2$
- Decreasing intercept

$$j(\nu, g) = 2 - \frac{4 + \nu^2}{2g} - \cdots$$

Weak 't Hooft Coupling I

- Amplitude $\mathcal{A}_{\text{planar}}(\sigma, \rho)$ computed to order g^4

$$-\frac{g^2}{2\pi^2}\Phi_1(z,\bar{z}) + \frac{g^4}{16\pi^4}\frac{2+2z\bar{z}-z-\bar{z}}{4z\bar{z}}\Phi_1^2(z,\bar{z}) \qquad z,\bar{z} = \sigma e^{\pm\rho}$$

$$+\frac{g^4}{16\pi^4}\frac{z\bar{z}}{z-\bar{z}}\left[\Phi_2(z,\bar{z}) - \Phi_2(1-z,1-\bar{z}) - \Phi_2\left(\frac{z}{z-1},\frac{\bar{z}}{\bar{z}-1}\right)\right]$$

with

$$\Phi_1 = \frac{z\bar{z}}{z-\bar{z}}\left[2\mathrm{H}_2 - \log z\bar{z}\,\mathrm{H}_1\right] \qquad \mathrm{H}_p(z,\bar{z}) = \mathrm{Li}_p(z) - \mathrm{Li}_p(\bar{z})$$

$$\Phi_2 = 6\mathrm{H}_4 - 3\log z\bar{z}\,\mathrm{H}_3 + \frac{1}{2}\log^2 z\bar{z}\,\mathrm{H}_2$$

Weak 't Hooft Coupling II

- In scattering regime obtain spin 1 Regge pole

$$\mathcal{A}_{\text{planar}} \simeq -\frac{g^4}{8\pi^2}\frac{\rho^2}{\sinh^2(\rho)} \qquad (\sigma \to 0)$$

corresponding to

$$j(\nu, g) \simeq 1 \qquad (g \to 0)$$

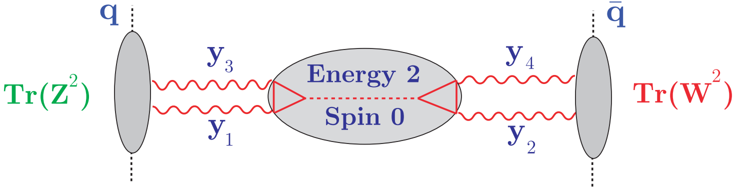
$$\alpha(\nu, g) \simeq -i\,\frac{\pi g^4}{16}\frac{\sinh\left(\frac{\pi\nu}{2}\right)}{\nu\cosh^3\left(\frac{\pi\nu}{2}\right)}$$

Small g and BFKL Pomeron Exchange I

- High energy hadron scattering (with $s \gg |t| \gg \Lambda_{\text{QCD}}$) at weak g dominated by hard perturbative Pomeron exchange
  - Quantum numbers of the vacuum 
  - Two–gluon color singlet state with ladder interactions (reggeized gluons)
  - Spin ~ 1 for $g \to 0$

$$j(\nu, g) = 1 + \frac{g^2}{4\pi^2}\left(2\Psi(1) - \Psi\left(\frac{1+i\nu}{2}\right) - \Psi\left(\frac{1-i\nu}{2}\right)\right) + \cdots$$

- BFKL picture for $\mathcal{N} = 4$ SYM for $g \to 0$



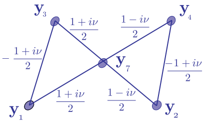
Small g and BFKL Pomeron Exchange II

- Amplitude ($y_i \in \partial$M gluon positions in transverse 2d space)

$$\mathcal{A} \sim \int_{\partial \mathrm{H}_3} \frac{dy_1 dy_3 dy_2 dy_4}{y_{13}^4 y_{24}^4} \; V(q, y_1, y_3) \; F(y_i) \; \bar{V}(\bar{q}, y_2, y_4)$$

- Pomeron propagator $F(y_i)$ sum of transverse partial waves of spin n and energy $|n-1|+1$. Only $n=0$ term contributes for scalar amplitudes

$$\int d\nu \, \frac{\nu^2}{(1+\nu^2)^2} \cdot$$

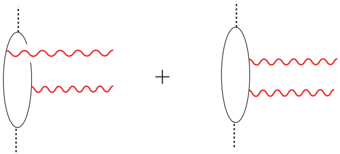


- Impact factor $V(q, y_1, y_3)$ constrained by conformal symmetry to be function of single cross ratio

$$u = \frac{-x^2 y_{13}^2}{(-2x \cdot y_1)(-2x \cdot y_3)}$$

Small g and BFKL Pomeron Exchange III

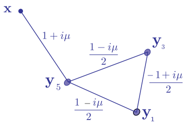
- Computable in perturbation theory



$$V \sim u^2 \left(1 - \delta\left(u\right)\right)$$

- Basis functions

$$\int d\mu \, V\left(\mu\right) \cdot \int_{\partial \mathrm{H}_3} dy_5 \cdot$$



- We obtain

$$\alpha\left(\nu\right) \simeq -\frac{i}{4\pi} V\left(\nu\right) \cdot \frac{\tanh \frac{\pi\nu}{2}}{\nu} \cdot \bar{V}\left(\nu\right)$$

with

$$V\left(\nu\right) = \bar{V}\left(\nu\right) = \frac{\pi g^2}{2} \frac{1}{\cosh \frac{\pi\nu}{2}}$$

Saturation at Weak Coupling I

- BFKL trajectory

$$j\,(\nu, g) = 1 + o(g^2)$$
$$\Delta\,(S, B) \;\text{ and }\; \beta\,(\nu) \;\text{ imaginary}$$

- Vanishing momentum transfer $t = 0$

$$s \qquad\qquad Q^2 \;\left(p_1^2 = p_3^2\right) \qquad\qquad \bar{Q}^2 \;\left(p_2^2 = p_4^2\right)$$

- Focus on cross section

$$\int \frac{dr}{r^3}\, f_1\,(r)\;\; f_3\,(r) \qquad\qquad r \sim 1/Q$$
$$\int \frac{d\bar{r}}{\bar{r}^3}\, f_2\,(\bar{r})\;\; f_4\,(\bar{r}) \qquad\qquad \bar{r} \sim 1/\bar{Q}$$
$$\int d^2\mathbf{b}\,\mathrm{Re}\left[1 - e^{2i\Delta(S,B)}\right] \qquad\qquad \sigma\,(s, Q, \bar{Q})$$

Saturation at Weak Coupling II

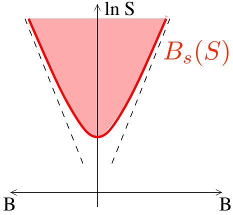
- Integral over impact parameter (with $S = s/Q\bar{Q}$)

$$\sigma(s, Q, \bar{Q}) \sim \frac{1}{Q\bar{Q}} \int_{|\ln Q/\bar{Q}|}^{\infty} dB \sinh B \,\mathrm{Re}\left[1 - e^{2i\Delta(S,B)}\right]$$

- For large B one has $|\Delta| \ll 1$. Phase shift ~ 1 along saturation line

$$\Delta \sim \int d\nu \, e^{\ln S (j(\nu)-1) - B(1+i\nu)}$$

so that



$$B_s(S) \sim \omega \ \ln S$$

$$\omega = 0.06\, g^2 + \cdots$$
$$\sim 0.14 \ \text{(exp. value)}$$

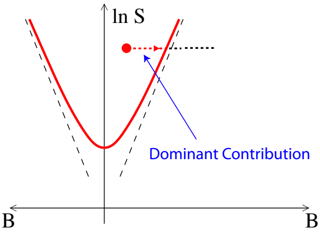
Saturation at Weak Coupling III

- Deep Saturation

$$|\ln Q/\bar{Q}| \lesssim B_s\,(s/Q\bar{Q})$$

- Approximate black disk

$$\frac{1}{Q\bar{Q}}\int_{|\ln Q/\bar{Q}|}^{B_s(S)} dB\,\sinh B\,\cdot\,1$$



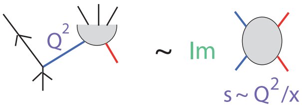
- When $B_s \sim \omega \ln S$ then

$$\frac{c}{Q\bar{Q}}\left[\left(\frac{s}{Q\bar{Q}}\right)^{\omega}+\left(\frac{s}{Q\bar{Q}}\right)^{-\omega}\right]-\frac{\tilde{c}}{Q\tilde{\Lambda}}\left[\frac{Q}{\tilde{\Lambda}}+\frac{\tilde{\Lambda}}{Q}\right]$$

with $c, \tilde{c}, \tilde{\Lambda}$ from $r, \bar{r}$ integrals

Applications to DIS I

- $\mathcal{O}_1$ E&M current (photon) $\mathcal{O}_2$ proton



- Kinematics

$$s \simeq Q^2/x$$

$\bar{Q}$ related to confinement scale &mass of proton

(simulate confinement with wavefunction in $\bar{r}$)

- Cross section for small x is

$$Q^{-2} F_2\left(x, Q^2\right)$$

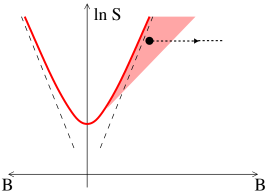
Geometric Scaling I

- Near saturation

$$|\ln Q/\bar{Q}| \gtrsim B_s\,(s/Q\bar{Q})$$

cross section reads

$$\int d^2\mathbf{b}\,\mathrm{Im}\,\Delta\,(S,B) \simeq \frac{1}{Q\bar{Q}N^2}\int d\nu\,\beta\,(\nu)\,\left(\frac{s}{Q\bar{Q}}\right)^{j(\nu)-1}\left(\frac{Q}{\bar{Q}}\right)^{-i\nu}$$



- At saddle point

$$\sigma \sim \frac{1}{\bar{Q}^2}\,\tau^{-(1+i\nu_s)\frac{1-\omega}{2}}$$

$$\tau = \left(\frac{Q}{Q_s}\right)^2 \qquad \text{with} \qquad Q_s = \bar{Q}^2\left(\frac{1}{x}\right)^{\frac{2\omega}{1-\omega}}$$

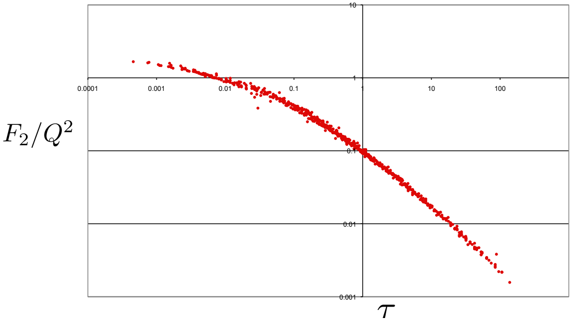
Geometric Scaling II

- In deep saturation

$$\sigma \sim \frac{1}{Q\bar{Q}} \left(\frac{Q}{x\bar{Q}} \right)^{\omega} \sim \frac{1}{\bar{Q}^2} \tau^{-\frac{1-\omega}{2}}$$

Specific dependence on the scaling variable

- Experimental evidence

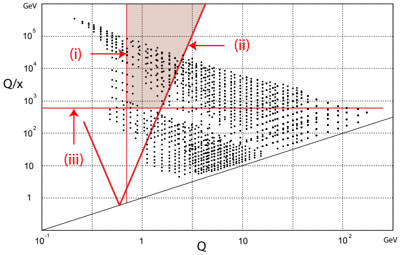


ω fixed experimentally to be $\omega \simeq 0.138 \pm 0.021$

Fit to DIS Data I

- Expression for $F_2\left(Q^2, x\right)$

$$\frac{cQ}{\bar{Q}}\left[\left(\frac{Q}{x\bar{Q}}\right)^{\omega}+\left(\frac{Q}{x\bar{Q}}\right)^{-\omega}\right]-\frac{\tilde{c}Q}{\tilde{\Lambda}}\left[\frac{Q}{\tilde{\Lambda}}+\frac{\tilde{\Lambda}}{Q}\right]$$



1. Weak coupling

$$Q > Q_{\mathrm{min}} \sim 0.7 - 1\ \mathrm{GeV}$$

2. Inside saturation

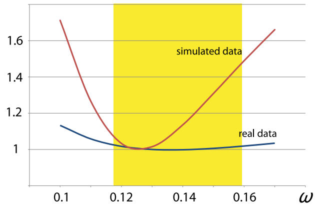
$$\omega \ln \frac{Q}{x\bar{Q}} > \ln \frac{Q}{\bar{Q}} \quad (\bar{Q} \sim 0.2 - 1\ \mathrm{GeV})$$

3. Asymptotic linear regime

$$\frac{Q}{x\bar{Q}} \gtrsim 10^{\eta} \qquad (\eta \gtrsim 3)$$

Fit to DIS Data II

- Minimize mean square deviation against experimental and simulated data

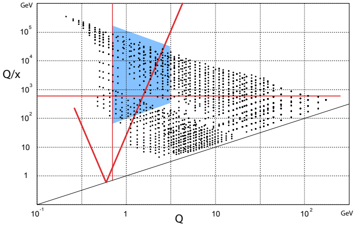


$$\omega \simeq 0.126$$

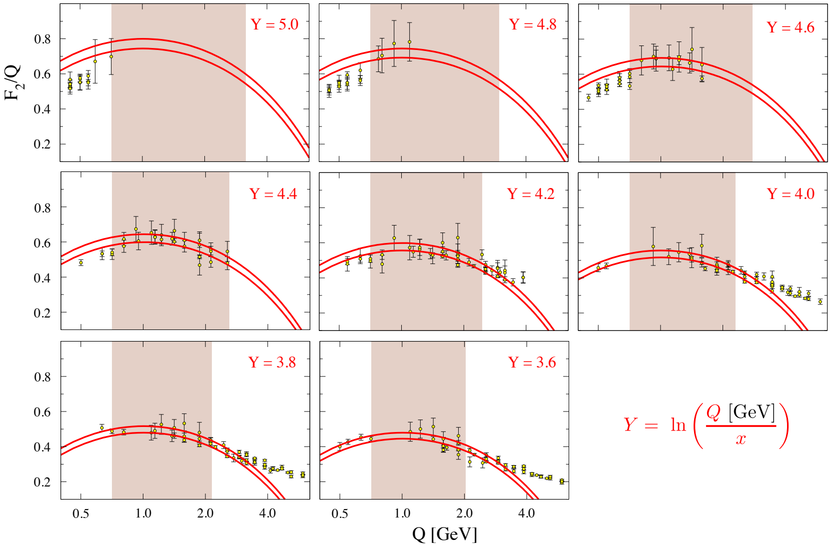
$$c \simeq 0.13$$
$$\tilde{c} \simeq 0.14$$
$$\tilde{\Lambda} \simeq 1 \text{ (GeV)}$$

- Match experimental data in rather large kinematical range with 6% accuracy

$$0.5 < Q^2 < 10$$
$$x < 10^{-2}$$

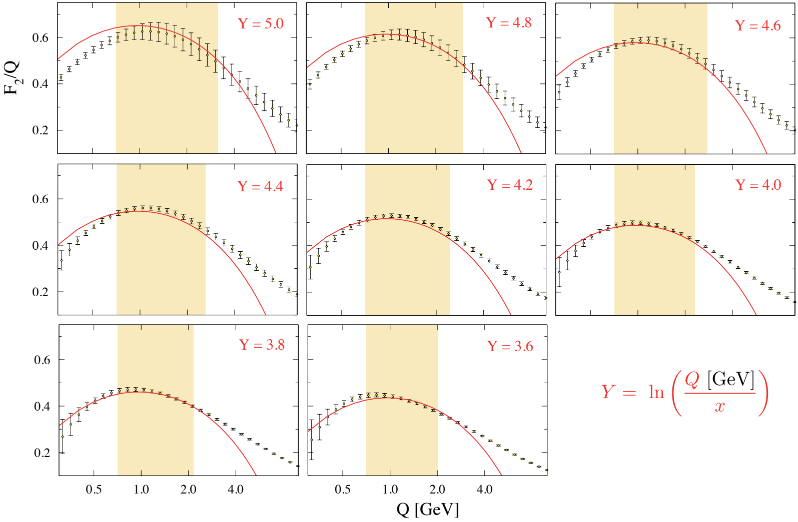


Real Data I



$$Y = \ln\left(\frac{Q\ [\mathrm{GeV}]}{x}\right)$$

Simulated Data I

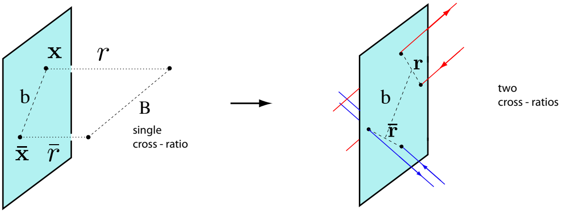


$$Y = \ln\left(\frac{Q\ [\text{GeV}]}{x}\right)$$

Comments on Dipole Formalism I

- Dipole phase shift

$$\Delta\left(s, r, \bar{r}, \mathbf{b}\right) \quad \rightarrow \quad \Delta_D\left(s, \mathbf{r}, \bar{\mathbf{r}}, \mathbf{b}\right)$$



- Representation of Δ_D

$$\int d\nu\, \alpha\left(\nu\right)\; s^{j(\nu)-1} \cdot \mathcal{T}_{i\nu}\left(\mathbf{r}, \bar{\mathbf{r}}, \mathbf{b}\right)$$

where

$$\mathcal{T}_{i\nu}\left(\mathbf{r}, \bar{\mathbf{r}}, \mathbf{b}\right) \simeq \left(|\mathbf{r}|\,|\bar{\mathbf{r}}|\,/\mathbf{b}^2\right)^{1+i\nu} \quad \text{for} \quad |\mathbf{r}|\,,|\bar{\mathbf{r}}| \ll |\mathbf{b}|$$

Comments on Dipole Formalism II

- For $r, \bar{r} \ll |\mathbf{b}|$

$$\begin{aligned} B &\sim \ln \mathbf{b}^2 / r\bar{r} \\ \Delta &\sim \int d\nu \, S^{j(\nu)-1} \cdot (r\bar{r}/\mathbf{b})^{1+i\nu} \end{aligned}$$

- Δ_D does not satisfy unitarity constraints (no asymptotic dipole states)
- Even if one assumes saturation at $\text{Im}\Delta_D \sim 1$, a simple exponential saddling for Δ_D is not possible for general $\mathbf{r}, \bar{\mathbf{r}}, \mathbf{b}$
- For $|\mathbf{r}|, |\bar{\mathbf{r}}| \ll |\mathbf{b}|$ one obtains only the first term in σ. A pure black disk is then a poor approximation of experimental data

Impact Factors for Spin 1 Operators I

- AdS graviton trajectory corresponds to $n = 0$ BFKL trajectory. What about $n \geq 1$ trajectories ?
- Consider as external states spin 1 operators

$$\mathcal{O}_1^A \qquad \mathcal{O}_2^A$$

- Impact factor

$$V^{mn}(q, y_1, y_3)$$

  with
  - symmetric in m, n and y_1, y_3
  - vanishing weight in q , y_1 , y_3
- Full amplitude

$$\frac{1}{|q|^{2\Delta_1} |\bar{q}|^{2\Delta_2}} \cdot \int_{\partial \mathrm{H}_3} \frac{dy_1 dy_3 dy_2 dy_4}{y_{13}^4 y_{24}^4} \; V^{mn}(q, y_1, y_3) \; F(y_i) \; \bar{V}^{\bar{m}\bar{n}}(\bar{q}, y_2, y_4)$$

Impact Factors for Spin 1 Operators II

- Transverse $SO(3,1)$ conformal symmetry implies

$$V^{mn} = \sum_{i=1}^{5} f_i(u)\ F_i^{mn}$$

with

$$\begin{aligned}
F_1^{mn} &= \eta^{mn} \\
F_2^{mn} &= \frac{q^m q^n}{q^2} \\
F_3^{mn} &= -\frac{1}{2} q^m \left(\frac{y_1^n}{q \cdot y_1} + \frac{y_3^n}{q \cdot y_3} \right) - \frac{1}{2} q^n \left(\frac{y_1^m}{q \cdot y_1} + \frac{y_3^m}{q \cdot y_3} \right) \\
F_4^{mn} &= -\frac{q^2}{4} \frac{y_1^m y_1^n}{(q \cdot y_1)^2} - \frac{q^2}{4} \frac{y_3^m y_3^n}{(q \cdot y_3)^2} \\
F_5^{mn} &= \frac{y_1^m y_3^n + y_3^m y_1^n}{y_{13}}
\end{aligned}$$

Impact Factors for Spin 1 Operators III

- Conserved current with $\Delta_1 = 3$ and

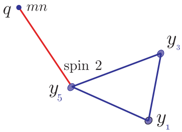
$$\frac{\partial}{\partial q^m}\left(\frac{1}{q^6} V^{mn}\right) = 0$$

- Projection on $n = 0$ and $n = 2$ trajectory

$$V^{mn} = V_0^{mn} + V_2^{mn}$$

- Construct $n = 2$ part using basis functions

$$V_2^{mn} \sim \int d\nu\, T(\nu) \int_{\partial H_3} dy_5 \cdot$$



- Spin 2 propagator from q to y_5 is unique and one can verify that

$$\frac{\partial}{\partial q^m} V_2^{mn} = 0 \qquad q_m V_2^{mn} = 0 \qquad V_2^{mn}\eta_{mn} = 0$$

Impact Factors for Spin 1 Operators IV

- Remaining four structures are $n = 0$ terms

$$V_0^{mn} = \sum_{i=1}^{4} \mathcal{D}_i^{mn} \, S_i\left(u\right)$$

with

$$\begin{aligned}
\mathcal{D}_1^{mn} &= \eta^{mn} - \frac{q^m q^n}{q^2} \\
\mathcal{D}_2^{mn} &= \frac{q^m q^n}{q^2} \\
\mathcal{D}_3^{mn} &= q^m \frac{\partial}{\partial q^n} + q^n \frac{\partial}{\partial q^m} \\
\mathcal{D}_4^{mn} &= q^2 \frac{\partial^2}{\partial q^m \partial q^n} + \left(q^m \frac{\partial}{\partial q^n} + q^n \frac{\partial}{\partial q^m} \right) - \frac{1}{3} \left(\eta^{mn} - \frac{q^m q^n}{q^2} \right) q^2 \Box_q
\end{aligned}$$

Impact Factors for Spin 1 Operators V

- Easy to determine S_1, S_2, S_3

$$\begin{aligned} S_1 &= f_1 + \frac{1}{6} f_4 + \frac{1-2u}{6} f_5 \\ S_2 &= f_1 + f_2 - 2f_3 - \frac{1}{2} f_4 - \frac{1}{2u} f_5 \\ S_3 &= \int \frac{du}{4u^2} \left(2uf_3 + uf_4 + f_5\right) \end{aligned}$$

- Complex to disentangle S_4 and the $n = 2$ contribution

$$V_\perp^{mn} = \mathcal{D}_4^{mn} S_4 + V_2^{mn}$$

with

$$q_m V_\perp^{mn} = 0 \qquad\qquad V_\perp^{mn} \eta_{mn} = 0$$

Impact Factors for Spin 1 Operators VI

- Use $\partial_m V_2^{mn} = 0$ to determine

$$(\Delta - 3)\, S_4 = \int \frac{du}{4u^2}\ \left(3uf_4 + 5f_5 + u^2(3u-2)f_4' + u(u-2)f_5'\right)$$

with

$$\Delta = 4u^2\,(1-u)\,\frac{d^2}{du^2} - 4u^2\frac{d}{du}$$

- To determine V_2^{mn} note that $V_\perp^{mn}$ can be viewed as an infinitesimal variation of the metric on H_3. The $n = 0$ term is then a combined diffeomorphism and Weyl transformation. Therefore, the infinitesimal Cotton tensor

$$C_{abc}\,(q, y_1, y_3)$$

due to a metric fluctuation $V_\perp^{mn}$ will only come from the $n = 2$ term

Impact Factors for Spin 1 Operators VII

- Due to conformal invariance and the symmetries of the Cotton tensor, we may construct a single Cotton function

$$C\left(u\right)=\frac{\left(-q^2\right)^2}{\left(-2y_1\cdot q\right)y_{13}}y_1^a y_3^b y_1^c C_{abc}$$

given explicitly by

$$\begin{aligned}C &= (1-u)u^2\left((3u-1)f_4'+u(3u-2)f_4''+\frac{u^2}{2}(u-1)f_4'''-\right.\\ &\left.-f_5'+(1-2u)f_5''-\frac{u}{2}(u-1)f_5'''\right)\end{aligned}$$

- From C one can immediately deduce $T\left(\nu\right)$

Examples in $\mathcal{N} = 4$ SYM I

- Basic spin 1 operators with $\Delta_1 = 3$ in the free limit

$$\mathrm{Tr}\left(\bar{\psi}\gamma_m\psi\right)$$
$$\mathrm{Tr}\left(\phi^i \overleftrightarrow{D}_m \phi^j\right) + c\ \mathrm{Tr}\left(\bar{\psi}\Gamma^{ij}\gamma_m\psi\right)$$
$$\mathrm{Tr}\left(\phi^i \overleftrightarrow{D}_m \phi^j\right) + c'\ \mathrm{Tr}\left(\bar{\psi}\Gamma^{ij}\gamma_m\psi\right)$$

- Only the R–symmetry current in the 15 of $SO(6)$ is chiral and has protected dimension. The other spin 1 operators (in the 15 and 1 of $SO(6)$) acquire ∞ dimension in the $g^2 \to \infty$ limit
- Very simple computation (compared to standard momentum space techniques) allows to compute
  - Impact factor for scalar quark current $\mathrm{Tr}\left(\bar{\phi} \overleftrightarrow{D}_m \phi\right)$

$$-3u^2\ F_4^{mn} + 2u^3\ F_5^{mn}$$

Examples in $\mathcal{N} = 4$ SYM II

- Impact factor for quark current $\mathrm{Tr}\left(\bar{\psi}\gamma_m\psi\right)$

$$2u^3\ F_1^{mn} + 4u^3\ F_5^{mn}$$

- The $n = 0$ contributions are different. The $n = 2$ contributions are identical with Cotton function

$$C = 36u^3\left(1 - u\right)\left(1 - 2u\right)$$

  Reinserting the $SO\left(6\right)$ factors one has that the R–symmetry current has no overlap with the $n = 2$ trajectory

- Basic Conjecture (tested already in more cases) : The SUGRA chirally protected states in $\mathcal{N} = 4$ SYM interact uniquely with the $n = 0$ Pomeron / graviton trajectory