

The BFKL Pomeron in AdS/CFT

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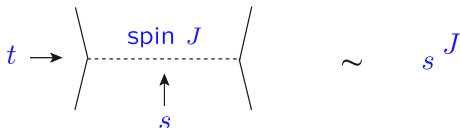
The IST String Fest – Lisbon, June 2009

Introduction I

- High energy phenomena probe relevant features of interactions
- In flat space string theory they are controlled by the leading Regge trajectory of states
- In perturbative YM theory (QCD and susy completions) dominated by Pomeron exchange
- High energy strings in holographic backgrounds will interpolate between these two regimes
 - We will present a general formalism valid both at weak and at strong coupling
 - At large 't Hooft coupling, analyze gravi-Reggeon exchange in AdS and recover flat space interactions
 - At weak coupling, use holography to correctly analyze known pQCD results

High Energy Scattering in Flat Space I

- Scattering in flat space in the Regge limit $s \gg -t$
- Single spin J exchange in the T -channel



Maximal spin dominates

- J unbounded \rightarrow High s behavior controlled by leading Regge pole in the complex J plane

$$\mathcal{A}_{\text{tree}}(s, t) \sim \beta(t) \cdot s^{j(t)}$$

- Strings in flat space

$$j(t) = 2 + \frac{\alpha'}{2} t$$

- Violation of unitarity at ultra-high energies

High Energy Scattering in Flat Space II

- Partial wave decomposition in the S -channel

$$\mathcal{A} \sim \sum_{J \geq 0} C_J \left(\frac{t}{s}\right) e^{2i\delta_J(s)} \quad (\text{Im}\delta_J(s) \geq 0)$$

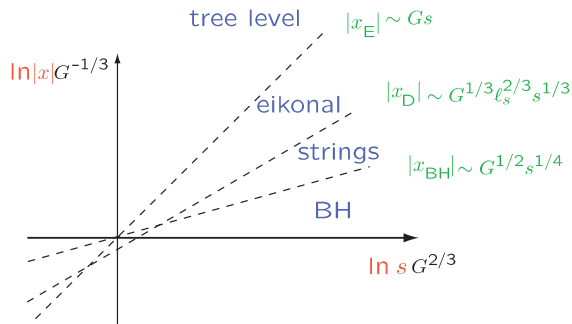
- In the Regge limit

$$\mathcal{A} \sim 2s \int d^3x e^{iq \cdot x + 2i\delta(s,x)} \quad \left(\begin{array}{l} q^2 = -t \\ |x| = 2J/\sqrt{s} \end{array} \right)$$

- Eikonal resummation of tree interaction

$$\mathcal{A}_{\text{tree}} \sim 4is \int d^3x e^{iq \cdot x} \delta(s,x)$$

High Energy Scattering in Flat Space III



- Eikonal limit when phase shift $\mathcal{A}_{\text{tree}}(s, x) / s \sim Gs / |x| \sim 1$. Multi gravi-reggeon exchange of **linear gravitons**
- In AdS_5 we expect similar physics with
 - Extra scale of the AdS radius ℓ
 - Transverse space H_3 instead of \mathbb{E}^3

High Energy Interactions in AdS I

- AdS₅ given by

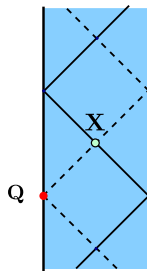
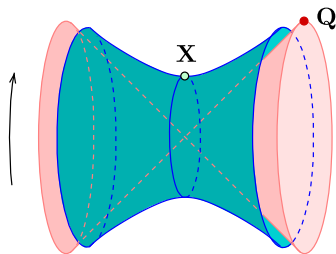
$$\mathbf{x}^2 = -\ell^2 \quad (\ell = 1)$$

$$\mathbf{x} \in \mathbb{M}^2 \times \mathbb{M}^4$$

Boundary points given by rays

$$Q^2 = 0$$

$$Q \sim \lambda Q$$



High Energy Interactions in AdS II

- Boundary points behave as momenta for AdS interactions. External wave functions

$$\frac{1}{(-2\mathbf{X} \cdot \mathbf{Q})^\Delta} \sim \int d\lambda e^{i\mathbf{X} \cdot \lambda \mathbf{Q}} \lambda^{\Delta-1}$$

dual to CFT operator of dimension Δ

- Scalar amplitudes

$$A(\mathbf{Q}_i)$$

depend on invariants

$$\mathbf{Q}_{ij} = -(\mathbf{Q}_i + \mathbf{Q}_j)^2$$

and are homogeneous

$$A(\dots, \lambda \mathbf{Q}_i, \dots) = \lambda^{-\Delta_i} A(\dots, \mathbf{Q}_i, \dots)$$

High Energy Interactions in AdS III

- Consider CFT correlator

$$\langle \mathcal{O}_1(\mathbf{Q}_1) \mathcal{O}_1^*(\mathbf{Q}_3) \mathcal{O}_2(\mathbf{Q}_2) \mathcal{O}_2^*(\mathbf{Q}_4) \rangle = \frac{1}{\mathbf{Q}_{13}^{\Delta_1} \mathbf{Q}_{24}^{\Delta_2}} \mathcal{A}(\text{cross ratios})$$

- Cross ratios

$$\frac{\mathbf{Q}_{13} \mathbf{Q}_{24}}{\mathbf{Q}_{12} \mathbf{Q}_{34}}$$

$$\frac{\mathbf{Q}_{14} \mathbf{Q}_{23}}{\mathbf{Q}_{12} \mathbf{Q}_{34}}$$

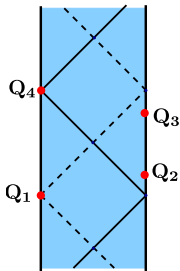
- Regge kinematics

$$\mathbf{Q}_{13}, \mathbf{Q}_{24} \rightarrow 0$$

- Basic example in $\mathcal{N} = 4$ SYM

$$\mathcal{O}_1 = \text{Tr}(Z^2) \quad \mathcal{O}_2 = \text{Tr}(W^2)$$

$$\mathcal{A} = 1 + N^{-2} \mathcal{A}_{\text{planar}} + \dots$$



T Channel View I

- Conventions

$$\mathbf{x} = (X^+, X^-, \mathbf{x}) \in \mathbb{M}^2 \times \mathbb{M}^4$$

$$\mathbf{x} = (x^+, x^-, \mathbf{x}) \in \mathbb{M}^4$$

- Scattering kinematics

$$\mathbf{Q}_1 = (0, 1, 0)$$

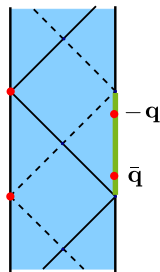
$$\mathbf{Q}_3 = -(q^2, 1, -q)$$

$$\mathbf{Q}_2 = (1, \bar{q}^2, \bar{q})$$

$$\mathbf{Q}_4 = -(1, 0, 0)$$

with

$$q, \bar{q} \in \text{Future light cone} \subset \mathbb{M}^4$$



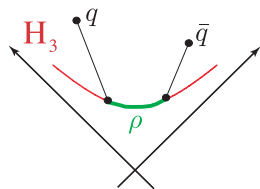
T Channel View II

- Transverse conformal group

$$\begin{array}{ll} SO(3,1) & q, \bar{q} \text{ vectors} \\ SO(1,1) & q, \bar{q} \rightarrow \lambda q, \lambda^{-1} \bar{q} \end{array}$$

- Cross ratios

$$\sigma^2 = q^2 \bar{q}^2 \qquad \sigma \cosh \rho = -q \cdot \bar{q}$$

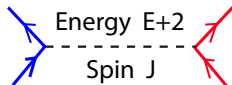


- Regge kinematics

$$\sigma \rightarrow 0 \qquad \rho \text{ fixed}$$

T Channel View III

- T-channel exchange $\mathcal{T}_{E,J}(\sigma, \rho)$ of a conformal block of



- Euclidean OPE for $\sigma \rightarrow 0$

$$\mathcal{T}_{E,J} \sim \sigma^{2+E}$$

- Scattering regime

$$\mathcal{T}_{E,J} \sim \sigma^{1-J} \Pi_E(\rho)$$

$\Pi_E(\rho)$ propagator of energy $1 + E$ in H_3

- General spin J contribution to correlator

$$\sigma^{1-J} \int dv \alpha_J(v) \Omega_{iv}(\rho) \quad \left(-\square_{H_3} = 1 + v^2 \right)$$

- Maximal spin dominates. If spin J unbounded resum contributions. Leading Regge pole at $J = j(\nu)$ gives

$$\mathcal{A}_{\text{planar}} \sim \int d\nu \sigma^{1-j(\nu)} \alpha(\nu) \Omega_{i\nu}(\rho)$$

- In $\mathcal{N} = 4$ SYM

$$\begin{array}{l} j(\nu, g) \\ \alpha(\nu, g) \end{array} \quad \left(g^2 = g_{\text{YM}}^2 N \right)$$

with $j(\nu, g)$ the spin of the twist two operator of dimension $2 + i\nu$

S Channel Eixonal Resummation I

- Impact parameter representation

$$\mathcal{A} \sim |q\bar{q}|^4 \int_{\mathcal{M}} dx d\bar{x} e^{-iq \cdot x - i\bar{q} \cdot \bar{x}} e^{2i\Delta(x, \bar{x})}$$

- Cross ratios

$$S = |x| |\bar{x}|$$

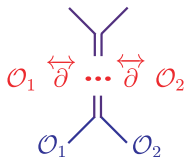
B impact parameter on H_3

- No interaction

$$\mathcal{A} = 1$$

$$\Delta = 0$$

- Lattice sum



$$1 = \sum_{E, J} \mathcal{S}_{E, J}$$

$$E = \Delta_1 + \Delta_2 + J + 2n$$

S Channel Eixonal Resummation II

- For large E, J replace Σ with $\int_{\mathbb{M}} dx d\bar{x}$ where

$$E = \sqrt{S} \cosh(B/2) \qquad J = \sqrt{S} \sinh(B/2)$$

- Phase shift determined eikonally by planar amplitude

$$\Delta \sim \frac{1}{N^2} \int dv S^{j(v)-1} \beta(v) \Omega_{iv}(B)$$

with

$$\alpha(v) = V_{\min}(v, j(v)) \cdot \beta(v) \cdot \bar{V}_{\min}(v, j(v))$$

- High energy unitarity

$$\text{Im } \Delta(S, B) \geq 0$$

- Anomalous dimension $-2\Delta/\pi$ of double trace $\mathcal{O}_1 \partial \cdots \partial \mathcal{O}_2$ of dimension E and spin J for elastic AdS interactions

Momentum Space I

- Correlator in momentum space

$$\langle \mathcal{O}_1(p_1) \mathcal{O}_1^*(p_3) \mathcal{O}_2(p_2) \mathcal{O}_2^*(p_4) \rangle$$

with

$$t, p_i^2 \text{ fixed} \quad s \rightarrow \infty$$

- AdS Poincaré coordinates

$$x^+, x^-, \underbrace{\mathbf{x}, r}_{H_3}$$

- External states

$$e^{-i\sqrt{s}x^-} e^{i\mathbf{p}_1 \cdot \mathbf{x}} f_1(r) \quad \left(r^2 \sim 1/p_1^2 \right)$$

- Amplitude

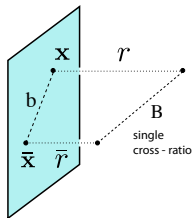
$$s \int \frac{dr d^2\mathbf{x}}{r^3} f_1(r) f_3(r) e^{i\mathbf{p} \cdot \mathbf{x}} \cdot \int \frac{d\bar{r} d^2\bar{\mathbf{x}}}{\bar{r}^3} f_2(\bar{r}) f_4(\bar{r}) e^{-i\mathbf{p} \cdot \bar{\mathbf{x}}} \cdot e^{2i\Delta(S,B)}$$

Momentum Space II

- Phase shift Δ depends on

$$S = sr\bar{r}$$

$$\cosh B = \frac{1}{2r\bar{r}} \left[r^2 + \bar{r}^2 + (\mathbf{x} - \bar{\mathbf{x}})^2 \right]$$



- Final answer

$$s \int d^2\mathbf{b} e^{i\mathbf{p}\cdot\mathbf{b}} e^{2i\delta(s,\mathbf{b})}$$

with

$$e^{2i\delta(s,\mathbf{b})} = s \int \frac{dr}{r^3} f_1(r) f_3(r) \cdot \int \frac{d\bar{r}}{\bar{r}^3} f_2(\bar{r}) f_4(\bar{r}) \cdot e^{2i\Delta(S,B)}$$

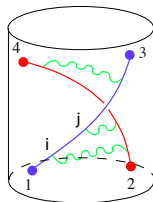
Large 't Hooft Coupling I

- Gravitational interaction in AdS₅ with

$$G \sim \frac{\ell^3}{N^2}$$


- Along \mathcal{O}_1 trajectory high energy states follow null geodesics in AdS parameterized by affine parameter x^+ and labelled by x^- and a point \mathbf{x}, r in H_3 with propagator

$$\frac{1}{\sqrt{s}} \theta(x_j^+ - x_i^+) \cdot \delta(x_j^- - x_i^-) \delta_{H_3}(\mathbf{x}_j, r_j | \mathbf{x}_i, r_i)$$



Large 't Hooft Coupling II

- For graviton exchange phase given by tree level interaction between geodesics

$$\Delta(S, B) = i \int dx^+ d\bar{x}^- \text{  } \simeq \frac{G}{\ell^3} S \Pi_2(B)$$

with $\Pi_2(B)$ propagator in H_3 of scalar dimension 3

- For $g^2 \rightarrow \infty$

$$j(v, g) \sim 2$$
$$\beta(v, g) \simeq \frac{\pi^2}{4 + v^2}$$

- Anomalous dimension of double trace $\mathcal{O}_1 \partial \cdots \partial \mathcal{O}_2$ of large dimension E and spin J

$$\frac{1}{4N^2} \frac{(E - J)^4}{EJ} \quad \left(\text{Exact in } 1/N^2 \right)$$

Including String Effects I

- Regge trajectory of the graviton $j(\nu, g)$ with

$$g^2 = \frac{\ell^4}{\alpha'^2} = g_{\text{YM}}^2 N$$

- Flat space limit

$$\lim_{\ell \rightarrow \infty} j(\ell\sqrt{-t}, \ell^2/\alpha') = 2 + \frac{\alpha'}{2}t$$

- Energy momentum tensor $j(\pm 2i, g) = 2$
- Decreasing intercept

$$j(\nu, g) = 2 - \frac{4 + \nu^2}{2g} - \dots$$

Weak 't Hooft Coupling I

- Amplitude $\mathcal{A}_{\text{planar}}(\sigma, \rho)$ computed to order g^4

$$-\frac{g^2}{2\pi^2} \Phi_1(z, \bar{z}) + \frac{g^4}{16\pi^4} \frac{2 + 2z\bar{z} - z - \bar{z}}{4z\bar{z}} \Phi_1^2(z, \bar{z}) \quad z, \bar{z} = \sigma e^{\pm i\rho}$$
$$+ \frac{g^4}{16\pi^4} \frac{z\bar{z}}{z - \bar{z}} \left[\Phi_2(z, \bar{z}) - \Phi_2(1 - z, 1 - \bar{z}) - \Phi_2\left(\frac{z}{z-1}, \frac{\bar{z}}{\bar{z}-1}\right) \right]$$

with

$$\Phi_1 = \frac{z\bar{z}}{z - \bar{z}} [2H_2 - \log z\bar{z} H_1] \quad H_p(z, \bar{z}) = \text{Li}_p(z) - \text{Li}_p(\bar{z})$$

$$\Phi_2 = 6H_4 - 3\log z\bar{z} H_3 + \frac{1}{2}\log^2 z\bar{z} H_2$$

Weak 't Hooft Coupling II

- In scattering regime obtain spin 1 Regge pole

$$\mathcal{A}_{\text{planar}} \simeq -\frac{g^4}{8\pi^2} \frac{\rho^2}{\sinh^2(\rho)} \quad (\sigma \rightarrow 0)$$

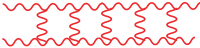
corresponding to

$$j(\nu, g) \simeq 1 \quad (g \rightarrow 0)$$

$$\alpha(\nu, g) \simeq -i \frac{\pi g^4}{16} \frac{\sinh\left(\frac{\pi\nu}{2}\right)}{\nu \cosh^3\left(\frac{\pi\nu}{2}\right)}$$

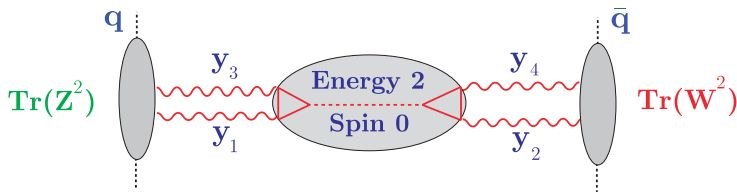
Small g and BFKL Pomeron Exchange I

- High energy hadron scattering (with $s \gg |t| \gg \Lambda_{\text{QCD}}$) at weak g dominated by hard perturbative Pomeron exchange

- Quantum numbers of the vacuum 
- Two-gluon color singlet state with ladder interactions (reggeized gluons)
- Spin ~ 1 for $g \rightarrow 0$

$$j(\nu, g) = 1 + \frac{g^2}{4\pi^2} \left(2\Psi(1) - \Psi\left(\frac{1+i\nu}{2}\right) - \Psi\left(\frac{1-i\nu}{2}\right) \right) + \dots$$

- BFKL picture for $\mathcal{N} = 4$ SYM for $g \rightarrow 0$



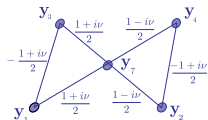
Small g and BFKL Pomeron Exchange II

- Amplitude ($y_i \in \partial M$ gluon positions in transverse 2d space)

$$\mathcal{A} \sim \int_{\partial H_3} \frac{dy_1 dy_3 dy_2 dy_4}{y_{13}^4 y_{24}^4} V(q, y_1, y_3) F(y_i) \bar{V}(\bar{q}, y_2, y_4)$$

- Pomeron propagator $F(y_i)$ sum of transverse partial waves of spin n and energy $|n-1|+1$. Only $n=0$ term contributes for scalar amplitudes

$$\int dv \frac{v^2}{(1+v^2)^2}$$

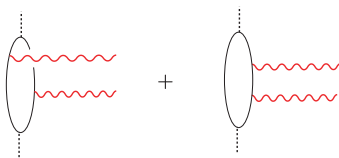


- Impact factor $V(q, y_1, y_3)$ constrained by conformal symmetry to be function of single cross ratio

$$u = \frac{-x^2 y_{13}^2}{(-2x \cdot y_1)(-2x \cdot y_3)}$$

Small g and BFKL Pomeron Exchange III

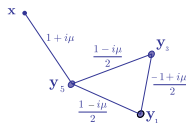
- Computable in perturbation theory



$$V \sim u^2 (1 - \delta(u))$$

- Basis functions

$$\int d\mu V(\mu) \cdot \int_{\partial H_3} dy_5 \cdot$$



- We obtain

$$\alpha(\nu) \simeq -\frac{i}{4\pi} V(\nu) \cdot \frac{\tanh \frac{\pi\nu}{2}}{\nu} \cdot \bar{V}(\nu)$$

with

$$V(\nu) = \bar{V}(\nu) = \frac{\pi g^2}{2} \frac{1}{\cosh \frac{\pi\nu}{2}}$$

Saturation at Weak Coupling I

- BFKL trajectory

$$j(\nu, g) = 1 + o(g^2)$$

$\Delta(S, B)$ and $\beta(\nu)$ imaginary

- Vanishing momentum transfer $t = 0$

$$s \qquad Q^2 \qquad \bar{Q}^2$$
$$\left(p_1^2 = p_3^2 \right) \qquad \left(p_2^2 = p_4^2 \right)$$

- Focus on cross section

$$\int \frac{dr}{r^3} f_1(r) f_3(r) \qquad r \sim 1/Q$$
$$\int \frac{d\bar{r}}{\bar{r}^3} f_2(\bar{r}) f_4(\bar{r}) \qquad \bar{r} \sim 1/\bar{Q}$$
$$\int d^2\mathbf{b} \operatorname{Re} \left[1 - e^{2i\Delta(S, B)} \right] \qquad \sigma(s, Q, \bar{Q})$$

Saturation at Weak Coupling II

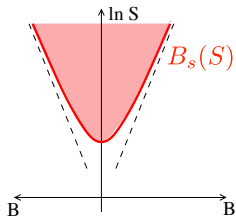
- Integral over impact parameter (with $S = s/Q\bar{Q}$)

$$\sigma(s, Q, \bar{Q}) \sim \frac{1}{Q\bar{Q}} \int_{|\ln Q/\bar{Q}|}^{\infty} dB \sinh B \operatorname{Re} \left[1 - e^{2i\Delta(S, B)} \right]$$

- For large B one has $|\Delta| \ll 1$. Phase shift ~ 1 along saturation line

$$\Delta \sim \int dv e^{\ln S(j(v)-1) - B(1+iv)}$$

so that



$$B_s(S) \sim \omega \ln S$$

$$\begin{aligned} \omega &= 0.06 g^2 + \dots \\ &\sim 0.14 \text{ (exp. value)} \end{aligned}$$

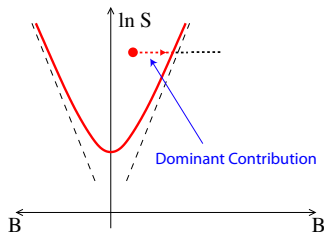
Saturation at Weak Coupling III

- Deep Saturation

$$|\ln Q/\bar{Q}| \lesssim B_s(s/Q\bar{Q})$$

- Approximate black disk

$$\frac{1}{Q\bar{Q}} \int_{|\ln Q/\bar{Q}|}^{B_s(S)} dB \sinh B \cdot \mathbf{1}$$



- When $B_s \sim \omega \ln S$ then

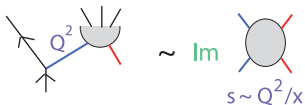
$$\frac{c}{Q\bar{Q}} \left[\left(\frac{s}{Q\bar{Q}} \right)^\omega + \left(\frac{s}{Q\bar{Q}} \right)^{-\omega} \right] - \frac{\tilde{c}}{Q\tilde{\Lambda}} \left[\frac{Q}{\tilde{\Lambda}} + \frac{\tilde{\Lambda}}{Q} \right]$$

with $c, \tilde{c}, \tilde{\Lambda}$ from r, \bar{r} integrals

Applications to DIS I

- \mathcal{O}_1 E&M current (photon)

\mathcal{O}_2 proton



- Kinematics

$$s \simeq Q^2/x$$

\bar{Q} related to confinement scale & mass of proton
(simulate confinement with wavefunction in \bar{r})

- Cross section for small x is

$$Q^{-2} F_2(x, Q^2)$$

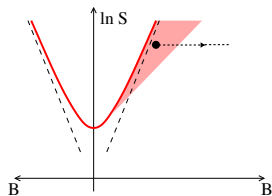
Geometric Scaling I

- Near saturation

$$|\ln Q/\bar{Q}| \gtrsim B_s (s/Q\bar{Q})$$

cross section reads

$$\int d^2\mathbf{b} \operatorname{Im} \Delta(S, B) \simeq \frac{1}{Q\bar{Q}N^2} \int dv \beta(v) \left(\frac{s}{Q\bar{Q}}\right)^{j(v)-1} \left(\frac{Q}{\bar{Q}}\right)^{-iv}$$



- At saddle point

$$\sigma \sim \frac{1}{\bar{Q}^2} \tau^{-(1+iv_s) \frac{1-\omega}{2}}$$

$$\tau = \left(\frac{Q}{Q_s}\right)^2 \quad \text{with} \quad Q_s = \bar{Q}^2 \left(\frac{1}{x}\right)^{\frac{2\omega}{1-\omega}}$$

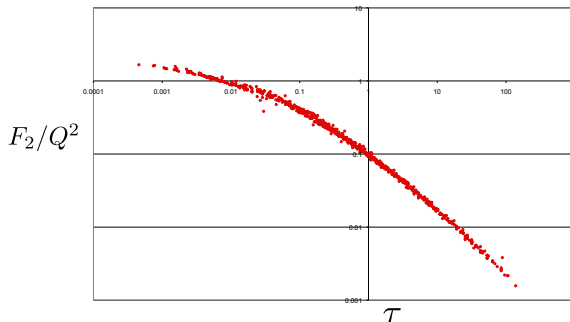
Geometric Scaling II

- In deep saturation

$$\sigma \sim \frac{1}{Q\bar{Q}} \left(\frac{Q}{x\bar{Q}} \right)^\omega \sim \frac{1}{\bar{Q}^2} \tau^{-\frac{1-\omega}{2}}$$

Specific dependence on the scaling variable

- Experimental evidence

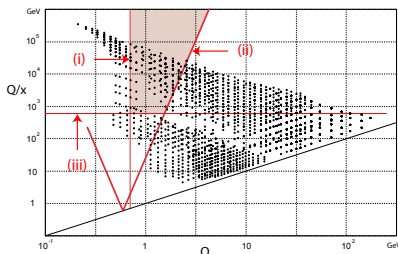


ω fixed experimentally to be $\omega \simeq 0.138 \pm 0.021$

Fit to DIS Data I

- Expression for $F_2(Q^2, x)$

$$\frac{cQ}{\bar{Q}} \left[\left(\frac{Q}{x\bar{Q}} \right)^\omega + \left(\frac{Q}{x\bar{Q}} \right)^{-\omega} \right] - \frac{\tilde{c}Q}{\tilde{\Lambda}} \left[\frac{Q}{\tilde{\Lambda}} + \frac{\tilde{\Lambda}}{Q} \right]$$



- 1 Weak coupling

$$Q > Q_{\min} \sim 0.7 - 1 \text{ GeV}$$

- 2 Inside saturation

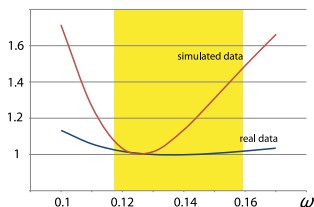
$$\omega \ln \frac{Q}{x\bar{Q}} > \ln \frac{Q}{\bar{Q}} \quad (\bar{Q} \sim 0.2 - 1 \text{ GeV})$$

- 3 Asymptotic linear regime

$$\frac{Q}{x\bar{Q}} \gtrsim 10^\eta \quad (\eta \gtrsim 3)$$

Fit to DIS Data II

- Minimize mean square deviation against experimental and simulated data



$$\omega \simeq 0.126$$

$$c \simeq 0.13$$

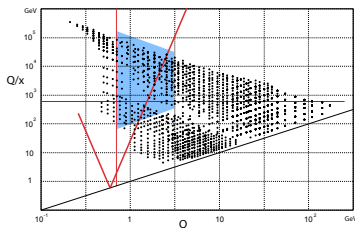
$$\tilde{c} \simeq 0.14$$

$$\tilde{\Lambda} \simeq 1 \text{ (GeV)}$$

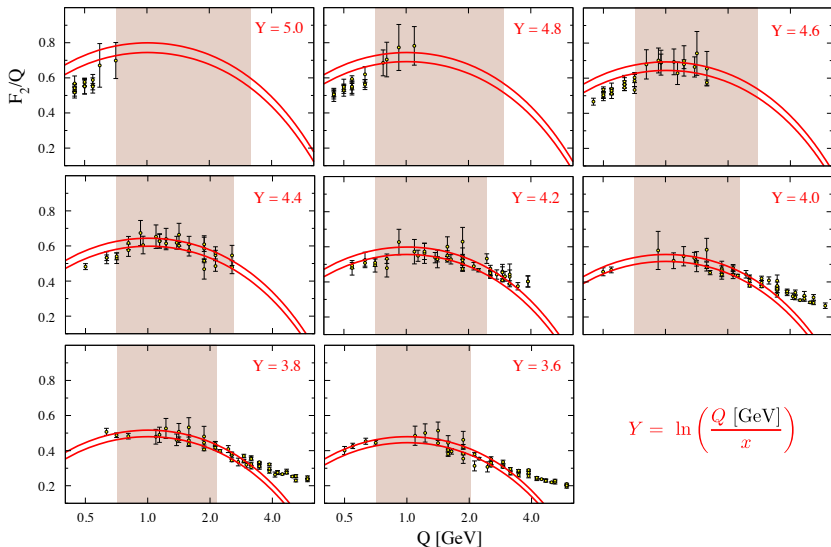
- Match experimental data in rather large kinematical range with 6% accuracy

$$0.5 < Q^2 < 10$$

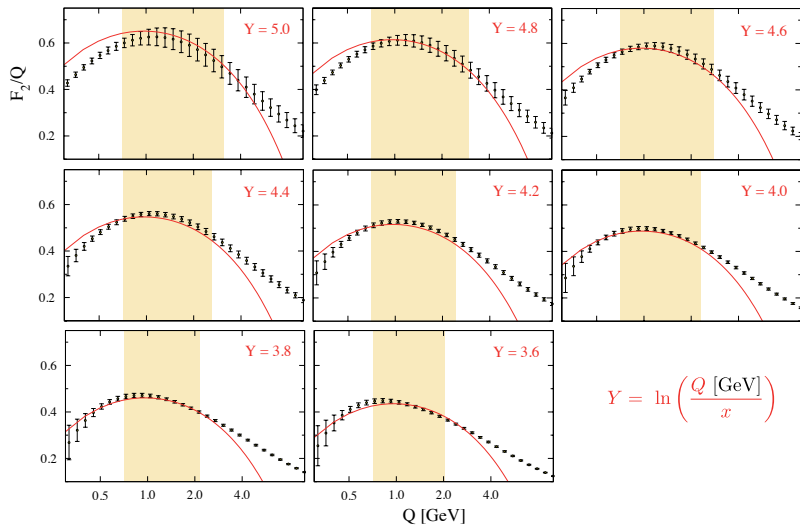
$$x < 10^{-2}$$



Real Data I



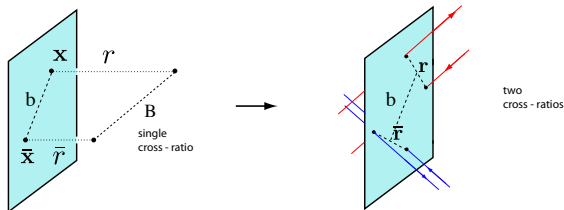
Simulated Data I



Comments on Dipole Formalism I

- Dipole phase shift

$$\Delta(s, r, \bar{r}, \mathbf{b}) \rightarrow \Delta_D(s, \mathbf{r}, \bar{\mathbf{r}}, \mathbf{b})$$



- Representation of Δ_D

$$\int d\nu \alpha(\nu) s^{j(\nu)-1} \cdot \mathcal{T}_{i\nu}(\mathbf{r}, \bar{\mathbf{r}}, \mathbf{b})$$

where

$$\mathcal{T}_{i\nu}(\mathbf{r}, \bar{\mathbf{r}}, \mathbf{b}) \simeq \left(|\mathbf{r}| |\bar{\mathbf{r}}| / \mathbf{b}^2 \right)^{1+i\nu} \quad \text{for} \quad |\mathbf{r}|, |\bar{\mathbf{r}}| \ll |\mathbf{b}|$$

Comments on Dipole Formalism II

- For $r, \bar{r} \ll |\mathbf{b}|$

$$B \sim \ln \mathbf{b}^2 / r \bar{r}$$

$$\Delta \sim \int d\nu S^{j(\nu)-1} \cdot (r \bar{r} / \mathbf{b})^{1+i\nu}$$

- Δ_D does **not** satisfy unitarity constraints (no asymptotic dipole states)
- Even if one assumes saturation at $\text{Im} \Delta_D \sim 1$, a **simple exponential saddling** for Δ_D is **not possible** for general $\mathbf{r}, \bar{\mathbf{r}}, \mathbf{b}$
- For $|\mathbf{r}|, |\bar{\mathbf{r}}| \ll |\mathbf{b}|$ one obtains **only the first term in σ** . A pure black disk is then a poor approximation of experimental data

Impact Factors for Spin 1 Operators I

- AdS graviton trajectory corresponds to $n = 0$ BFKL trajectory. What about $n \geq 1$ trajectories ?
- Consider as external states spin 1 operators

$$\mathcal{O}_1^A \quad \mathcal{O}_2^A$$

- Impact factor

$$V^{mn}(q, y_1, y_3)$$

with

- symmetric in m, n and y_1, y_3
- vanishing weight in q, y_1, y_3
- Full amplitude

$$\frac{1}{|q|^{2\Delta_1} |\bar{q}|^{2\Delta_2}} \cdot \int_{\partial H_3} \frac{dy_1 dy_3 dy_2 dy_4}{y_{13}^4 y_{24}^4} V^{mn}(q, y_1, y_3) F(y_i) \bar{V}^{\bar{m}\bar{n}}(\bar{q}, y_2, y_4)$$

Impact Factors for Spin 1 Operators II

- Transverse $SO(3,1)$ conformal symmetry implies

$$V^{mn} = \sum_{i=1}^5 f_i(u) F_i^{mn}$$

with

$$F_1^{mn} = \eta^{mn}$$

$$F_2^{mn} = \frac{q^m q^n}{q^2}$$

$$F_3^{mn} = -\frac{1}{2} q^m \left(\frac{y_1^n}{q \cdot y_1} + \frac{y_3^n}{q \cdot y_3} \right) - \frac{1}{2} q^n \left(\frac{y_1^m}{q \cdot y_1} + \frac{y_3^m}{q \cdot y_3} \right)$$

$$F_4^{mn} = -\frac{q^2}{4} \frac{y_1^m y_1^n}{(q \cdot y_1)^2} - \frac{q^2}{4} \frac{y_3^m y_3^n}{(q \cdot y_3)^2}$$

$$F_5^{mn} = \frac{y_1^m y_3^n + y_3^m y_1^n}{y_{13}}$$

Impact Factors for Spin 1 Operators III

- Conserved current with $\Delta_1 = 3$ and

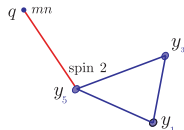
$$\frac{\partial}{\partial q^m} \left(\frac{1}{q^6} V^{mn} \right) = 0$$

- Projection on $n = 0$ and $n = 2$ trajectory

$$V^{mn} = V_0^{mn} + V_2^{mn}$$

- Construct $n = 2$ part using basis functions

$$V_2^{mn} \sim \int dv T(v) \int_{\partial H_3} dy_5 \cdot$$



- Spin 2 propagator from q to y_5 is unique and one can verify that

$$\frac{\partial}{\partial q^m} V_2^{mn} = 0$$

$$q_m V_2^{mn} = 0$$

$$V_2^{mn} \eta_{mn} = 0$$

Impact Factors for Spin 1 Operators IV

- Remaining four structures are $n = 0$ terms

$$V_0^{mn} = \sum_{i=1}^4 \mathcal{D}_i^{mn} S_i(u)$$

with

$$\mathcal{D}_1^{mn} = \eta^{mn} - \frac{q^m q^n}{q^2}$$

$$\mathcal{D}_2^{mn} = \frac{q^m q^n}{q^2}$$

$$\mathcal{D}_3^{mn} = q^m \frac{\partial}{\partial q^n} + q^n \frac{\partial}{\partial q^m}$$

$$\mathcal{D}_4^{mn} = q^2 \frac{\partial^2}{\partial q^m \partial q^n} + \left(q^m \frac{\partial}{\partial q^n} + q^n \frac{\partial}{\partial q^m} \right) - \frac{1}{3} \left(\eta^{mn} - \frac{q^m q^n}{q^2} \right) q^2 \square_q$$

Impact Factors for Spin 1 Operators V

- Easy to determine S_1, S_2, S_3

$$S_1 = f_1 + \frac{1}{6}f_4 + \frac{1-2u}{6}f_5$$

$$S_2 = f_1 + f_2 - 2f_3 - \frac{1}{2}f_4 - \frac{1}{2u}f_5$$

$$S_3 = \int \frac{du}{4u^2} (2uf_3 + uf_4 + f_5)$$

- Complex to disentangle S_4 and the $n = 2$ contribution

$$V_{\perp}^{mn} = \mathcal{D}_4^{mn} S_4 + V_2^{mn}$$

with

$$q_m V_{\perp}^{mn} = 0 \quad V_{\perp}^{mn} \eta_{mn} = 0$$

Impact Factors for Spin 1 Operators VI

- Use $\partial_m V_2^{mn} = 0$ to determine

$$(\Delta - 3) S_4 = \int \frac{du}{4u^2} \left(3uf_4 + 5f_5 + u^2(3u - 2)f_4' + u(u - 2)f_5' \right)$$

with

$$\Delta = 4u^2(1 - u) \frac{d^2}{du^2} - 4u^2 \frac{d}{du}$$

- To determine V_2^{mn} note that V_{\perp}^{mn} can be viewed as an infinitesimal variation of the metric on H_3 . The $n = 0$ term is then a combined diffeomorphism and Weyl transformation. Therefore, the infinitesimal Cotton tensor

$$C_{abc}(q, y_1, y_3)$$

due to a metric fluctuation V_{\perp}^{mn} will only come from the $n = 2$ term

Impact Factors for Spin 1 Operators VII

- Due to conformal invariance and the symmetries of the Cotton tensor, we may construct a single Cotton function

$$C(u) = \frac{(-q^2)^2}{(-2y_1 \cdot q) y_{13}} y_1^a y_3^b y_1^c C_{abc}$$

given explicitly by

$$C = (1-u)u^2 \left((3u-1)f_4' + u(3u-2)f_4'' + \frac{u^2}{2}(u-1)f_4''' - f_5' + (1-2u)f_5'' - \frac{u}{2}(u-1)f_5''' \right)$$

- From C one can immediately deduce $T(v)$

Examples in $\mathcal{N} = 4$ SYM I

- Basic spin 1 operators with $\Delta_1 = 3$ in the free limit

$$\text{Tr}(\bar{\psi}\gamma_m\psi)$$

$$\text{Tr}\left(\phi^i \overleftrightarrow{D}_m \phi^j\right) + c \text{Tr}\left(\bar{\psi}\Gamma^{ij}\gamma_m\psi\right)$$

$$\text{Tr}\left(\phi^i \overleftrightarrow{D}_m \phi^j\right) + c' \text{Tr}\left(\bar{\psi}\Gamma^{ij}\gamma_m\psi\right)$$

- Only the R-symmetry current in the 15 of $SO(6)$ is chiral and has protected dimension. The other spin 1 operators (in the 15 and 1 of $SO(6)$) acquire ∞ dimension in the $g^2 \rightarrow \infty$ limit
- Very simple computation (compared to standard momentum space techniques) allows to compute
 - Impact factor for scalar quark current $\text{Tr}\left(\bar{\phi} \overleftrightarrow{D}_m \phi\right)$

$$-3u^2 F_4^{mn} + 2u^3 F_5^{mn}$$

Examples in $\mathcal{N} = 4$ SYM II

- Impact factor for quark current $\text{Tr}(\bar{\psi}\gamma_m\psi)$

$$2u^3 F_1^{mn} + 4u^3 F_5^{mn}$$

- The $n = 0$ contributions are different. The $n = 2$ contributions are identical with Cotton function

$$C = 36u^3 (1 - u) (1 - 2u)$$

Reinserting the $SO(6)$ factors one has that the R-symmetry current has no overlap with the $n = 2$ trajectory

- **Basic Conjecture (tested already in more cases) :** The SUGRA chirally protected states in $\mathcal{N} = 4$ SYM interact uniquely with the $n = 0$ Pomeron / graviton trajectory