New approaches to higher-dimensional black holes

- Blackfolds and Domain Structure

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IST String Fest at Instituto Superior Tecnico
Lisbon, Portugal, June 29 – July 1, 2009

Based on
arXiv:0904.4246 ”Domain Structure of Black Hole Space-Times”
PRL 102: 191301 (2009) with Emparan, Niarchos and Obers
JHEP 0170:110 (2007) with Emparan, Niarchos, Obers and Rodriguez
and other less recent papers.

Motivation:

Why study black holes in higher dimensions?

► String Theory:

String Theory: Strong candidate for Quantum Gravity and unified theory of Nature

Critical superstring theory: D=10

- BHs in up to 10 space-time dimensions are part of String Theory

Extended objects in String Theory: D-branes, NS-branes, etc.

Many possible charged objects, give rise to many interesting objects with event horizons

Important to know finite temperature phases of string theory
- String theory models for the early universe

Important for understanding BH entropy in string theory
- Understanding of BH phases important for understanding underlying microstates

- ► Holography

  Phases of D > 4 black holes dual to phases of gauge theories at finite temperature

  AdS/CFT

  Possible to understand gauge theory phases using dual black holes

  Important for understanding dynamical properties in gauge theories,
  e.g. transport properties, universal viscosity bound found using BHs, etc.

  Black holes $\leftrightarrow$ Fluid dynamics (see Shiraz' talk)

- ► Black holes in LHC: Planck scale could be low, phenomenologically viable scenarios with large extra dimensions

- ► Better understanding of General Relativity, dimension D as tunable parameter

  Which properties are
  - Universal, e.g. laws of BH mechanics
  - D-dependent, e.g. uniqueness, topologies, shapes, stability

Outline of talk:

Motivation

Review of asymptotically flat black holes

The blackfold approach

Examples, Bestiary, etc.

Charged blackfolds

Summary

} First part of talk

Recap of last time

Review of rod-structure and 5D solutions

Domain structure

Conclusions

} Second part

Review of asymptotically flat black holes:

We focus (mostly) on **asymptotically flat** solutions of **pure gravity**:

$$R_{\mu\nu} = 0$$

What exact solutions are known?

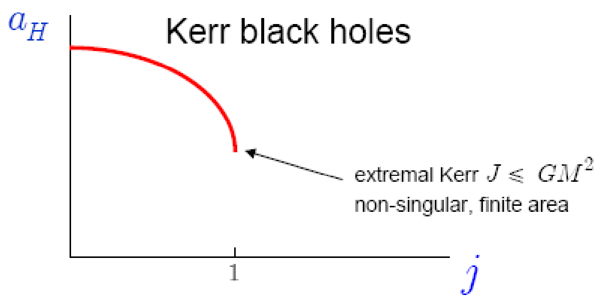
Usual D=4 General Relativity:

Things pretty simple: Only one available BH phase given mass M and ang. mom. J

- Schwarzschild BH: The unique static BH for a given mass M
- Kerr BH: The unique stationary BH for a given mass M and ang. mom. J

Schwarzschild and Kerr BHs have event horizon topology S^2

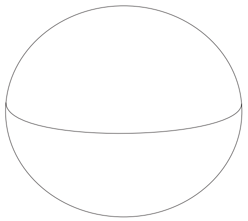
Schwarzschild and Kerr BHs are stable to perturbations



Kerr bound is $J \leq GM^2$

The minimal generalization of BHs for D>4 should include a BH with spherical topology S^{D-2}

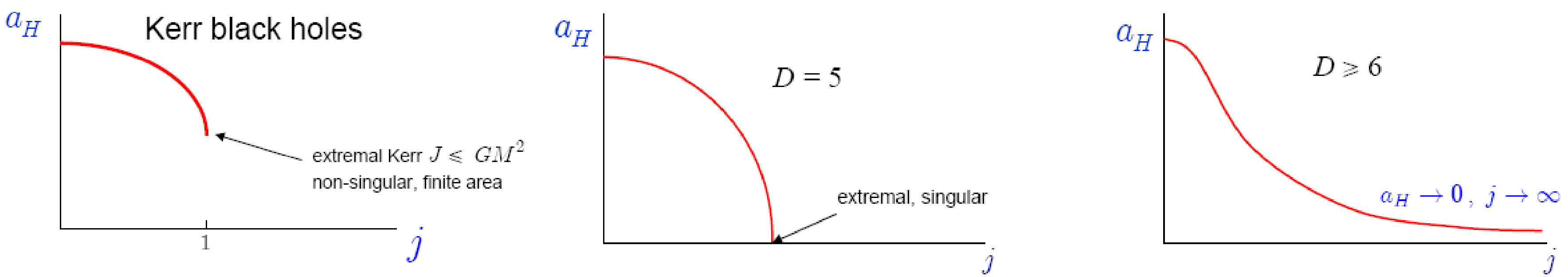
- Schwarzschild-Tangherlini BH (1963): D-dim. static spherically symmetric BH
- Myers-Perry BH (1986): D-dim. stationary BH



Event horizon topology S^{D-2}

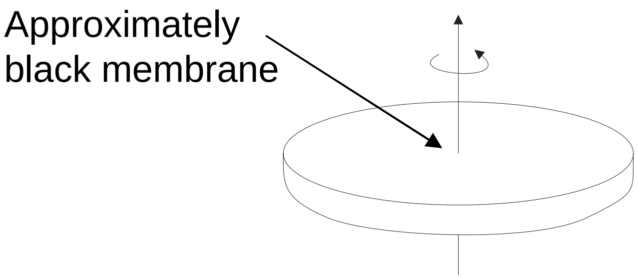
Uniqueness and stability of Schwarzschild-Tangherlini: Gibbons,Ida,Shiromizu Ishibashi,Kodama

Special feature for D $\geq$ 6 Myers-Perry BHs: You have ultraspinning regimes



Emparan & Myers considered the $J \to \infty$ ultraspinning limit of the Myers-Perry black hole for $D \geq 6$ (with only one ang. mom. turned on)

Result: For large J, the MP BH near the axis of rotation becomes approximately a static black membrane

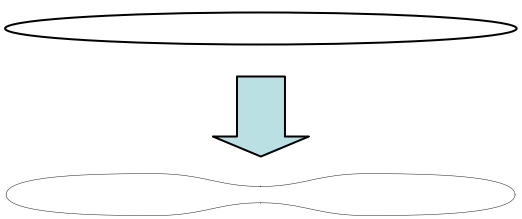


$\Rightarrow$ Supports a GL mode

$\Rightarrow$ The MP black hole is classically unstable for large J

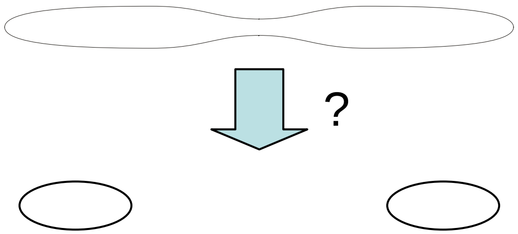
This type of instability is not there for small J

Ishibashi & Kodama



$\Rightarrow$ Conjecture: Pinched MP BH phase emerging at the critical value of J

What about:



This suggests that there is more to higher-dimensional BHs than just the Myers-Perry BH

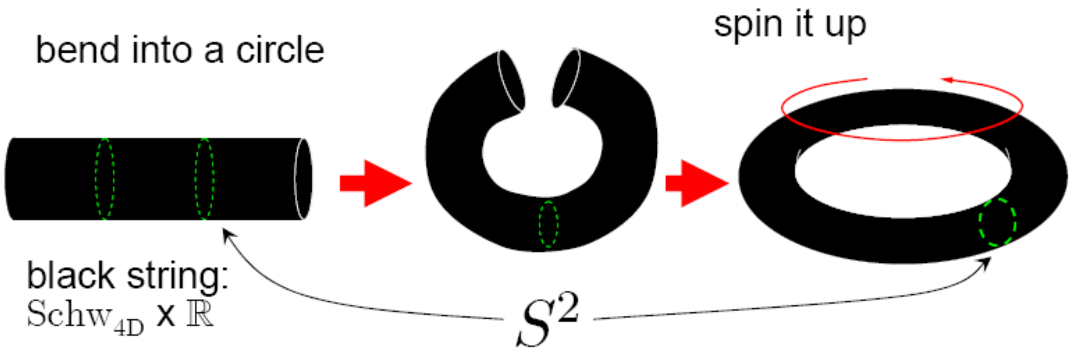
This would be a rotating black ring!

...We will come back to this...

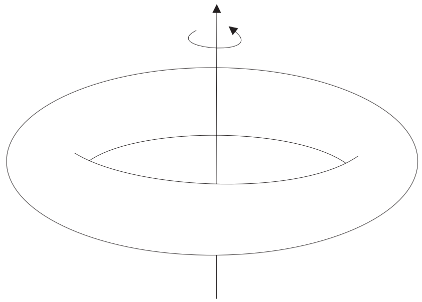
Should you expect more solutions than the Myers-Perry BH to be possible?

Yes: You have more freedom for higher D

One can imagine bending a black string into a ring, and let it rotate so that the rotation gives a solution in equilibrium



Emparan and Reall found an exact black ring solution for D=5 in 2001



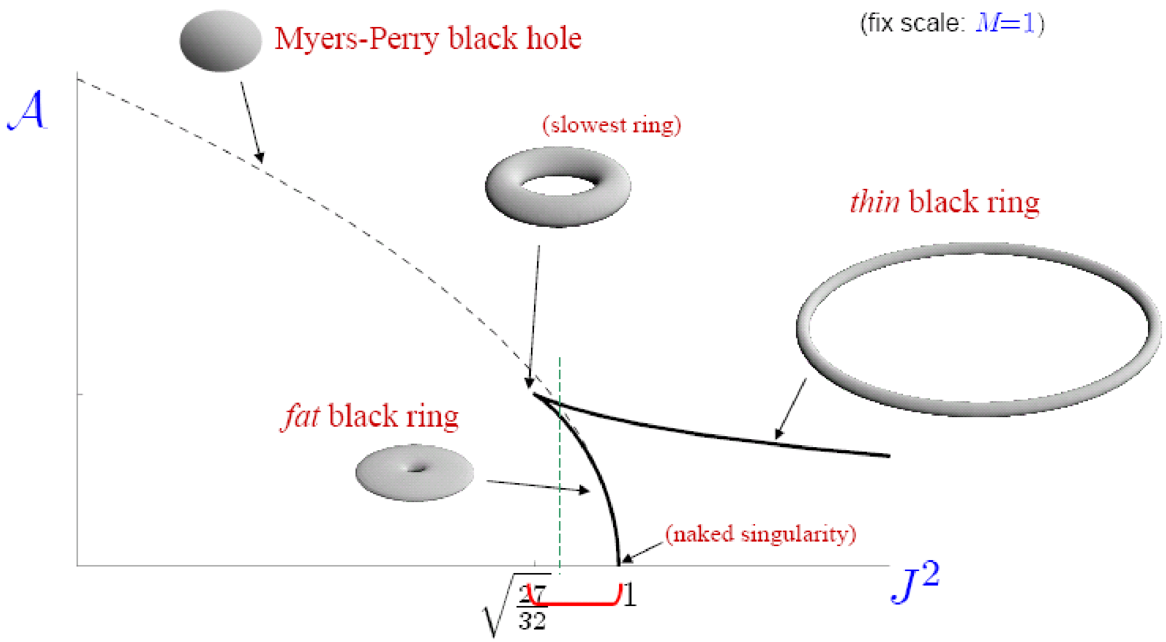
The regular black ring rotates along the ”ring-direction”

Horizon topology $S^1 \times S^2$

A static black ring solution is singular – needs extra force from the singularity to preserve equilibrium

Emparan-Reall Black ring was generalized to include two angular momenta by Pomeransky and Senkov (2006)

Phase diagram for D=5 stationary black holes:



Non-uniqueness of black holes in D=5 (Given asymptotic charges)!

Consequence: Uniqueness is not a generic property of black holes for general D - It is specific to D=4!

So how to classify BHs in higher dimensions? More on this in second part of talk

You can combine the Myers-Perry BH and black ring to produce multi-black hole solutions in D=5, more on this in second part of talk

Finding of exact 5D Black ring solution raises some important questions:

Why not a black ring for D $\geq$ 6?

What if we take a membrane and bend it to a torus? Or a sphere?

Why are the only the known exact BH solutions for D $\geq$ 6 the Myers-Perry BH and Schwarzschild-Tangherlini BH?

Low D: Symmetries provides strong constraints
We have powerful solution generating techniques using integrability
$\Rightarrow$ Few solutions, but it is easy to find them

High D: Symmetries are sparse, alot of freedom is available, no general solution generating technique exists
$\Rightarrow$ Many solutions, but it is hard to find them

D = 5: Combines low D and high D features: We have powerful constraints from symmetries but at the same time alot of freedom

We cannot expect to understand what BHs are possible for D $\geq$ 6 just from the study of exact solutions

We need an approach that does not rely on exact solutions

How do we understand what BHs are possible without knowing exact solutions?

Topological censorship:

In 4D: Hawking proved only S^2 topology possible for event horizon (assuming dominant energy condition)

Generalized by Galloway and Schoen to $D \geq 4$:

Horizon topology should be of positive Yamabe type

(it must admit a metric of positive curvature)

D=4: S^2

D=5: S^3, $S^1 \times S^2$, L(p,q)

D=6: S^4, $S^1 \times S^3$, $S^2 \times S^2$, $T^2 \times S^2$,...

D=5: It's consistent with the solutions found, but not constraining enough to give new insights

$D \geq 6$: Doesn't give much information on which of these horizons can be realized physically (and how)

Conclusion: Topological censorship is not sufficient

We need a more direct physical approach

The blackfold approach

The Blackfold approach:

Emparan, TH, Niarchos, Obers & Rodriguez
Emparan, TH, Niarchos & Obers

For any D-dim. asymptotically flat black hole define the two length scales

$$L_M = (GM)^{1/(D-3)} \qquad L_J = \frac{1}{M}\sqrt{\sum_i J_i^2}$$

Our key insights:

- For D $\geq$ 5 there are BHs for which it is possible to have $L_J \gg L_M$
- In the limit $L_M/L_J \to 0$ of such a BH the BH is a flat black brane near the horizon
- We can find effective description of the BH in this limit by integrating out the L_M scale d.o.f.'s

Not possible in D=4: Kerr bound $J \leq GM^2$ gives that $L_J \leq L_M$

The above can be seen as being responsible for the new dynamics of BHs as compared to 4D (i.e. that we have more than just the Myers-Perry BH)

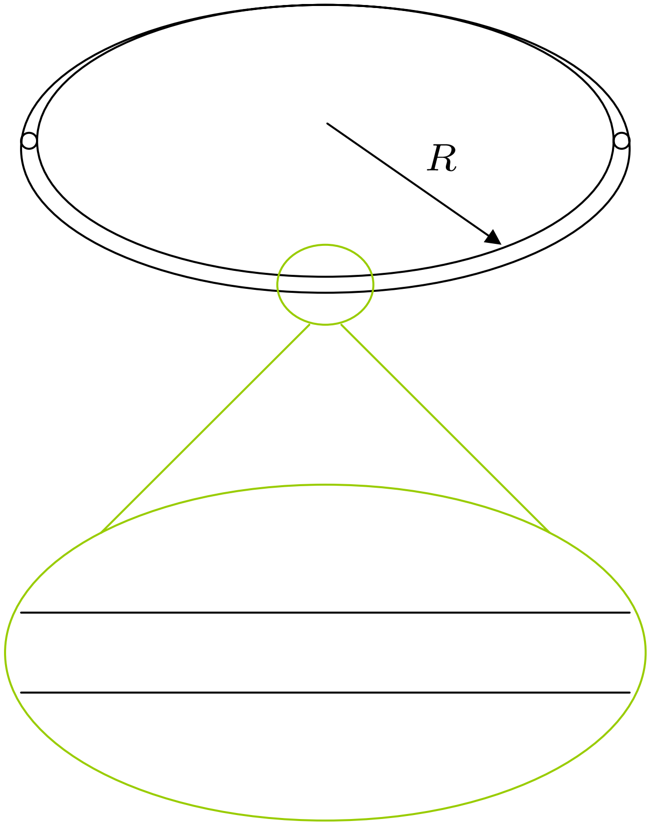
Also: Higher-dim. BHs should be organised according to the L_M,L_J scales

$L_J \lesssim L_M$: BHs behave qualitative like in 4D

$L_J \sim L_M$: Threshold of new dynamics

$L_J \gg L_M$: Blackfold regime

Examples of the $L_M/L_J \to 0$ limit:



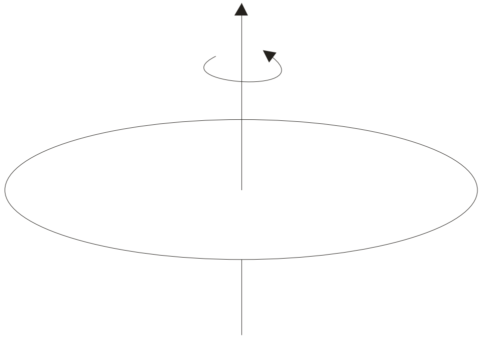
D=5: $r_0/R \to 0$ limit of Black ring

r_0 : S^2 radius

R : S^1 radius

A thin black ring limit

D $\geq$ 6: Ultraspinning limit of MP BH Emparan & Myers



MP BH flattens out to a disc (pancake)

Thickness of disc $\ll$ Radius of disc

Near center: Like a static flat black membrane

Idea of Blackfold approach:

Use the widely separate scales $L_J \gg L_M$ to construct BHs perturbatively by integrating out short-distance d.o.f.'s and make effective description in terms of the long-distance d.o.f.'s

To leading order this can be thought of as the **probe approximation**

We put a non-extremal black p-brane probe in a background geometry

$$I = I_{\text{bg}}[g_{\mu\nu}] + I_{\text{brane}}[g_{\mu\nu}, X^{\mu}, \lambda]$$

Einstein-Hilbert for pure gravity

The effective action for the long-distance d.o.f.'s

Analogous to DBI action for extremal D-branes

$X^{\mu}(\sigma^{\alpha})$: The embedding of the p-brane in the background

$\sigma^{\alpha} = (\sigma^0,...,\sigma^p)$: World-volume coordinates of the p-brane

λ : Further brane parameters (will be clear soon)

Take the background to be D-dimensional Minkowski space

We consider then a p-dimensional submanifold $\mathcal{B}_p$ of $\mathbb{R}^{D-1}$

The embedding of $\mathcal{B}_p$ in $\mathbb{R}^{D-1}$ is described by $X^\mu(\sigma)$ with $X^0(\sigma) = \sigma^0$

Blackfold: A **black** brane curved along a submani**fold**

Example: p=1, $\mathcal{B}_1 = S^1$, $\theta(\sigma) = \sigma^1$, $r(\sigma) = R \Rightarrow$ A black ring

What is the non-extremal p-brane probe action $I_{brane}[g_{\mu\nu}, X^\mu, \lambda]$?

Carter derived a mechanical equilibrium equation for any brane probe:

$$(\nabla_\alpha^{(h)} \partial_\beta X^\rho + \Gamma^\rho_{\mu\nu} \partial_\alpha X^\mu \partial_\beta X^\nu) \tau^{\alpha\beta} = 0$$

where the induced metric on the world-volume is $h_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X^\nu g_{\mu\nu}$

We see that this generalizes Newtons 2nd law and the geodesic eq. in GR

EM tensor $\tau^{\alpha\beta}$ is what distinguish a particular kind of brane probe

$\rightarrow$ We should put in $\tau^{\alpha\beta}$ for a non-extremal p-brane

$\tau^{\alpha\beta}$ the EM tensor of a black p-brane:

We demand that for each point of $\mathcal{B}_p$ we can make a local Lorentz transformation so that locally the EM tensor is that of a flat static black p-brane:

$$\tau_{00} = (n+1)r_0^n \qquad \tau_{ii} = -r_0^n \quad , \quad i = 1, 2, ..., p$$

r_0 : Schwarzschild radius of the black p-brane

We defined here n = D-3-p

Meaning of boosts on world-volume of p-brane:

Boost on flat brane world-volume $\leftrightarrow$ Angular rotation of BH (= target space)

Consider BH with m angular momenta turned on:

m rotation planes that we want the blackfold to rotate in

$$(r_i, \phi_i) \quad , \quad i = 1, 2, ..., m$$

We align m of the directions of the p-brane with the angular directions:

$\phi_i(\sigma) = \sigma^i$ for i=1,2,...,m (thus $m \leq p$)

These m directions, and the time direction, will be the directions in which we make local Lorentz transformations (boosts)

Boosts (Lorentz transformations): SO(1,m) $\subset$ SO(1,p)

Rotation invariance SO(m) of τ_{ab}
$\Rightarrow$ We only need m(m+1)/2 – m(m-1)/2 = m parameters to specify boost

$$\Lambda_0{}^0 = \cosh\alpha$$

$$\Lambda_i{}^0 = \nu_i \sinh\alpha \quad , \quad i = 1,2,...,m \quad \text{with} \quad \sum_{i=1}^{m} \nu_i^2 = 1$$

$$\Lambda_j{}^0 = 0 \quad , \quad j = m+1,...,p$$

Note that here we determined the EM tensor with flat indices τ_{ab} since we are using a local Lorentz frame to map with the flat black brane

Boosted EM tensor:

$$\tau_{00} = (n\cosh^2\alpha + 1)r_0^n$$

$$\tau_{ii} = (n\nu_i^2 \sinh^2\alpha - 1)r_0^n \qquad i = 1,2,...,m$$

$$\tau_{i\neq j} = n\nu_i\nu_j \sinh^2\alpha r_0^n \qquad i,j = 1,2,...,m$$

$$\tau_{0i} = n\nu_i \sinh\alpha \cosh\alpha r_0^n \qquad i,j = 1,2,...,m$$

$$\tau_{ii} = -r_0^n \quad , \quad i = m+1,...,p$$

For each point of the blackfold $\mathcal{B}_p$: We have the boost parameters α, ν_i

Thus $\alpha = \alpha(\sigma)$ and $\nu_i = \nu_i(\sigma)$ $\leftarrow$ boost parameters functions on the world-volume

So the blackfold is now locally a boosted black brane

But we still need to impose that it is in thermal equilibrium:

Blackness condition:

Surface gravity κ and angular velocities Ω_i constant on the blackfold

We can find κ and Ω_i locally in terms of embedding and $\tau^{\alpha\beta}$:

$$\kappa = \frac{n}{2 r_0 \cosh\alpha} \qquad \Omega_i = \frac{\nu_i}{r_i} \tanh\alpha \quad , \quad i = 1, 2, ..., m$$

From this and $\sum_{i=1}^m \nu_i^2 = 1$ we get

$$\cosh\alpha(\sigma) = \frac{1}{\sqrt{1 - \Xi(\sigma)^2}} \qquad \nu_i(\sigma) = \frac{r_i(\sigma)\Omega_i}{\Xi(\sigma)} \qquad r_0(\sigma) = \frac{n}{2\kappa}\sqrt{1 - \Xi(\sigma)^2}$$

(thickness of brane)

with $\Xi(\sigma) \equiv \sqrt{\sum_{i=1}^m (r_i(\sigma))^2 \Omega_i^2}$ ⟵ The velocity field

We see now that the EM tensor $\tau^{\alpha\beta}$ is completely determined by

$\kappa, \Omega_i, r_i(\sigma)$ ⟵ Depends on embedding (part of $X^\mu(\sigma)$)

Look again at the Carter equation:

$$(\nabla_\alpha^{(h)} \partial_\beta X^\rho + \Gamma^\rho_{\mu\nu} \partial_\alpha X^\mu \partial_\beta X^\nu) \tau^{\alpha\beta} = 0$$

This equation can now be written solely in terms of

$$\Omega_i, \ X^\mu(\sigma), \ \partial_\alpha X^\mu(\sigma), \ \partial_\alpha \partial_\beta X^\mu(\sigma)$$ (κ factorizes out)

Carter equation becomes a set of purely geometric equations for the embedding of $\mathcal{B}_p$ for given Ω_i

⇒ **Geometric censorship for blackfolds**

For example: A round S^1 satisfy Carter, but a wiggly S^1 doesn't

We can also now display the variables of the probe brane action:

$$I_{\text{brane}}[g_{\mu\nu}, X^\mu, \kappa, \Omega_i]$$

(κ can only enter in the normalization)

We determined the EOM's that this action should contain
Can we also determine the action itself?

Action principle for blackfolds:

We can find an action by writing down the thermodynamic potential for the Grand Canonical ensemble with variables κ and Ω_i

This is: $$I_{\text{thermo}}(\kappa, \Omega_i) = M - \sum_i \Omega_i J_i - 4\pi\kappa A_H$$

Using $$M = \int \sqrt{h}\tau_{00} \qquad J_i = \int \sqrt{h} r_i \tau_{0i} \qquad A_H = \int \sqrt{h} r_0^{n+1} \Omega_{n+1} \cosh\alpha$$

We get $$I_{\text{thermo}} = \left(\frac{n}{2\kappa}\right)^n \int \sqrt{h}\left[1 - \Xi(\sigma)^2\right]^{n/2}$$

Idea: Vary I_{thermo} wrt. $X^\mu(\sigma)$ keeping fixed κ and Ω_i

⇔ 1st law of thermodynamics ⇔ Geometric EOM's for blackfold

Thus, up to normalization we have:

($\mathcal{N}$ the normalization)

$$I_{\text{brane}} = \mathcal{N} \int \sqrt{h}\left(1 - \Xi(\sigma)^2\right)^{\frac{n}{2}}$$

This action contains Blackfold mechanics in full!

Zero tension law for blackfolds:

We can take the local Smarr formula for the flat black p-brane and integrate it up over the total blackfold – This gives:

$$(D-3)M = (D-2)(\Omega_i J_i + TS) + \mathcal{T}$$

where $\mathcal{T} = -\int \sqrt{h} \sum_{i=1}^{p} \tau_{ii}$ is the total integrated tension of the blackfold

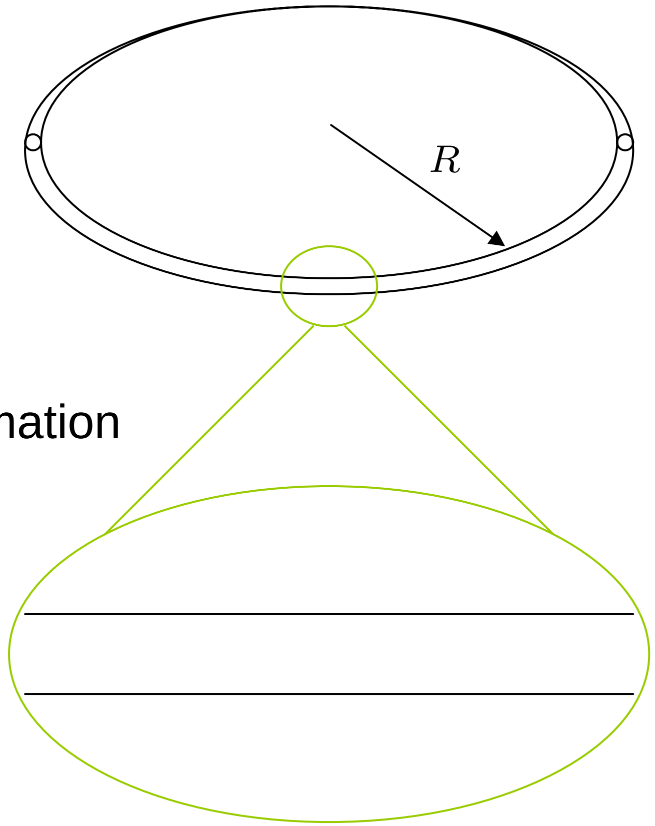
But for asymptotically flat solution with a single horizon we have in general the Smarr formula:

$$(D-3)M = (D-2)(\Omega_i J_i + TS)$$

Hence in general total tension $\mathcal{T} = 0$ $\leftarrow$ A zero tension law for blackfolds!

This we can check explicitly for the examples of blackfolds

Before turning to examples I would like to mention some work that the blackfold approach builds on:



The ”blackfold program” was started with a perturbative construction of black ring metric for D $\geq$ 5

Emparan, TH, Niarchos, Obers & Rodriguez

Here we went one step further than the probe approximation

$\Rightarrow$ We computed the first order correction to the metric near the black string horizon

$\Rightarrow$ Important check on regularity of horizon

Technique: Matched Asymptotic Expansion

Matched Asymptotic Expansion first developed for localized Kaluza-Klein BHs in the limit in which the size of the BH is much smaller than size of extra dimension

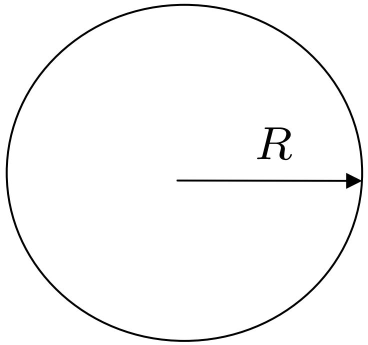
TH. Gorbonos & Kol. Karasik et al.

Powerful method based on field theory also developed

Chu, Goldberger & Rothstein. Kol.

Examples:

Topology: $S^1 \times S^{D-3}$



The black ring:

Embedding: S^1 in $\mathbb{R}^2$ (times a point in $\mathbb{R}^{D-3}$)

$\mathbb{R}^2$: (r,ϕ) $r = R(\sigma)$ $\phi = \sigma$

Action: $$I_{\text{brane}} \propto \int \sqrt{h}(1-\Xi^2)^{\frac{n}{2}} = \int \sqrt{R'^2 + R^2}(1-\Omega^2 R^2)^{\frac{n}{2}}$$

The full EOM from this is

$$(1-\Omega^2 R^2) R R'' + ((n+2)\Omega^2 R^2 - 2) R'^2 + ((n+1)\Omega^2 R^2 - 1) R^2 = 0$$

highly non-linear diff. eq.

But it has a simple solution: $$R = \frac{1}{\sqrt{n+1}} \frac{1}{\Omega}$$

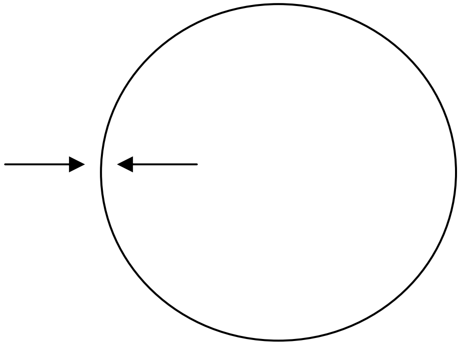
Geometric censorship:
Round rings allowed,
wiggly rings not

Can also be seen from direct variation of action with R=constant

Also from Carter eq.: $$K_{11}\tau^{11} = 0 \Rightarrow \frac{1}{R}\tau^{11} = 0 \Rightarrow \tau_{11} = 0$$

Total tension vanishes $\mathcal{T} = 0$

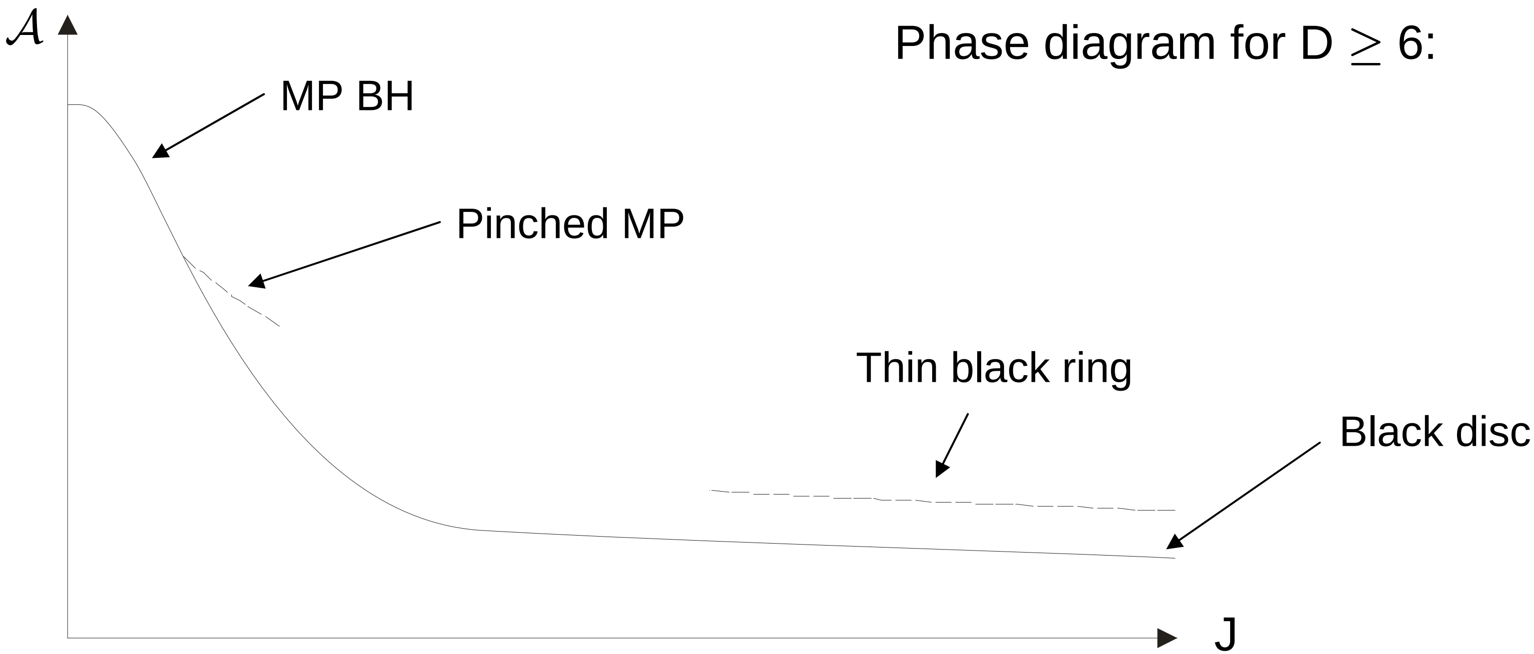
A zero tension condition $\Leftrightarrow$ Balance of forces on ring

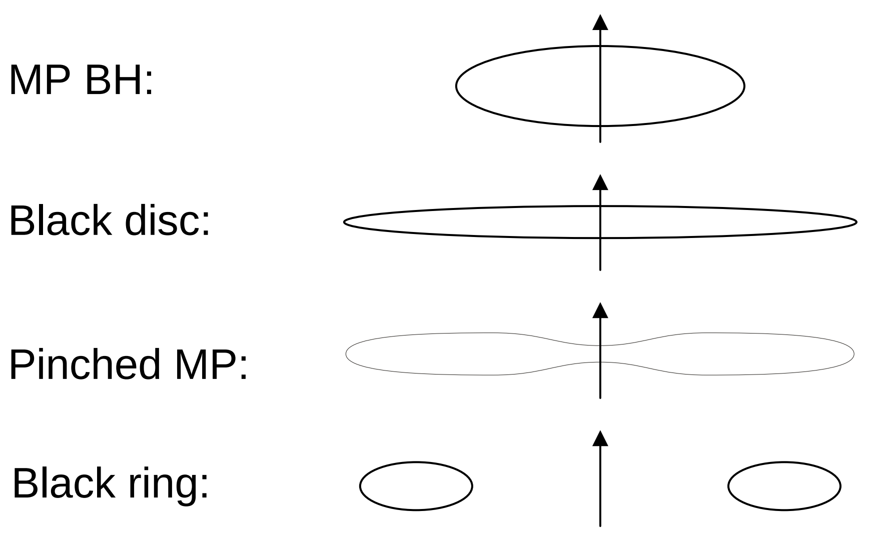


The ring should rotate fast enough to balance the tension of the black string

...Back to the question of the MP BH/Black ring phase diagram:

Phase diagram for D $\geq$ 6:





Thin black ring dominates entropically over ultraspinning Myers-Perry BH:

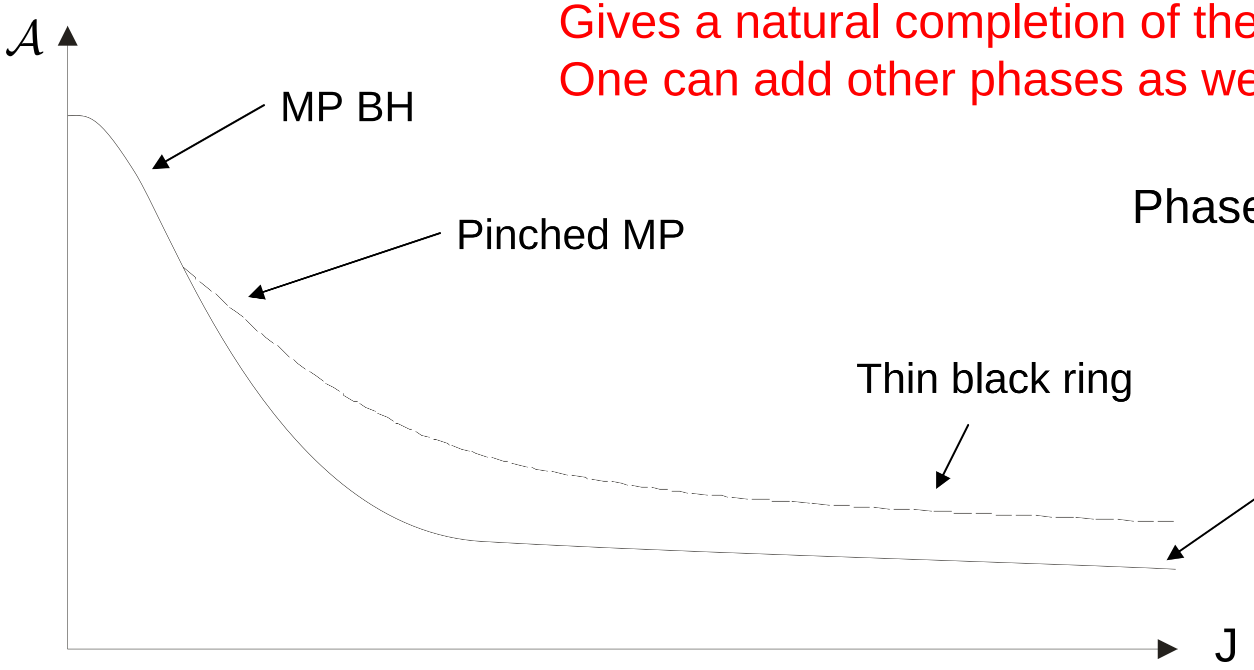
$$S_{\text{bh}}(M,J) \propto J^{-\frac{1}{D-4}} M^{\frac{D-2}{D-4}}$$

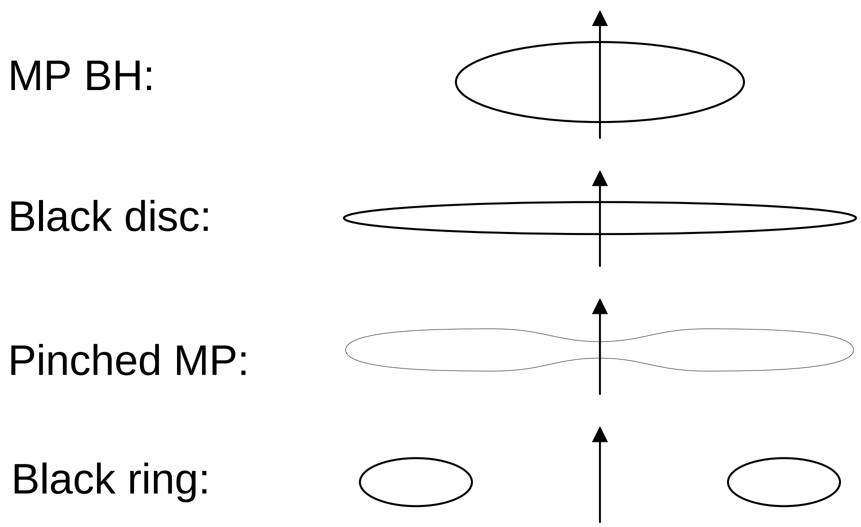
$$S_{\text{br}}(M,J) \propto J^{-\frac{2}{D-5}} M^{\frac{D-2}{D-5}}$$

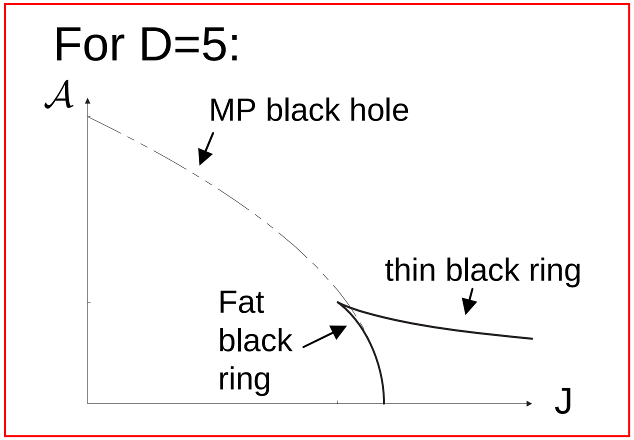
...It seems natural to connect the pinched MP and black ring phases like this:

Gives a natural completion of the phase diagram
One can add other phases as well (Black Saturn etc.)

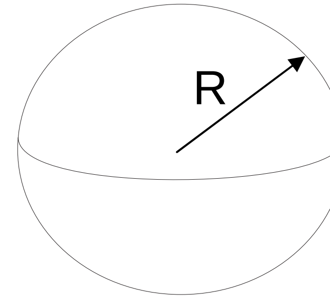
Phase diagram for D $\geq$ 6:







<u>Black odd-spheres</u>: $\mathcal{B}_{2k+1} = S^{2k+1}$



Embedding: S^{2k+1} in $\mathbb{R}^{2k+2}$ (times point in $\mathbb{R}^{D-3-2k}$)

$\mathbb{R}^{2k+2}$: k+1 planes: (r_i, ϕ_i) , $i = 1, 2, ..., k+1$

parameterize this as

$$r_i = \mu_i R \quad , \quad i = 1, 2, ..., k+1 \qquad \text{with} \quad \sum_{i=1}^{k+1} \mu_i^2 = 1$$

If we assume R=constant and all Ω_i equal the action is

$$I_{\text{brane}} \propto \int R^p (1 - \Omega^2 R^2)^{\frac{n}{2}}$$

EOM solved by $R = \sqrt{\frac{p}{n+p} \frac{1}{\Omega}}$

This is equivalent to: $\sum_{i=1}^{p} \tau_{ii} = 0$ ⟵ Zero-tension condition, gives $\mathcal{T} = 0$

Same as Carter eq., zero force condition

Horizon topology: $S^{2k+1} \times S^{D-3-2k}$

For $k \geq 1$: The boosts depend on location on the S^{2k+1}

Products of odd-spheres:

$$\mathcal{B}_p = \prod_{a=1}^{k} S^{p_a} \qquad p_a = \text{odd} \qquad \sum_{a=1}^{k} p_a = p$$

Constraint on number of rotation planes:

$$k + p = \sum_{a=1}^{k} (1 + p_a) \leq D - 1 \Leftrightarrow k \leq n + 2$$

The action (constant sphere radius for each sphere, same Ω_a for each)

$$I_{\text{brane}} \propto \prod_a \int R_a^{p_a} (1 - \Omega_a^2 R_a^2)^{\frac{n}{2}}$$

Solution:
$$R_a = \sqrt{\frac{p_a}{n+p}} \frac{1}{\Omega_a}$$

Examples: T^p, $S^3 \times T^{p-3}$, $S^3 \times S^3$, etc.

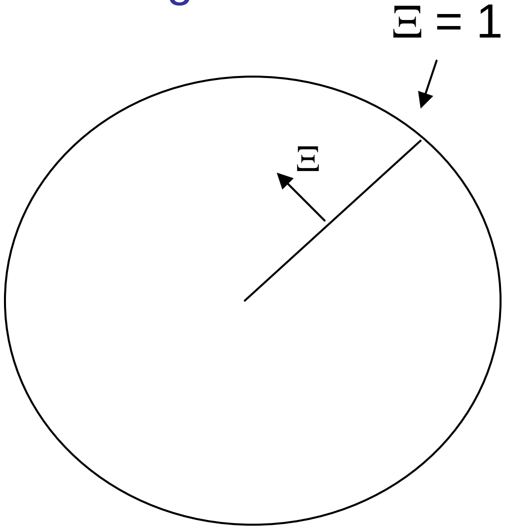
Black disc: Consider the plane: $ds^2 = dr^2 + r^2 d\phi^2$

Embed blackfold $\mathcal{B}_2$ as follows $r(\sigma^1, \sigma^2) = \sigma^1 \quad , \quad \phi(\sigma^1, \sigma^2) = \sigma^2$

Blackness condition requires constant Ω

Hence the velocity field is $\Xi = r\Omega \Rightarrow$ Rigid rotation around the origin

Blackfold equations solved for any value of Ω



When $r = 1/\Omega$ we have $\Xi = 1$: Velocity of light

From $r_0(\sigma) = \frac{n}{2\kappa}\sqrt{1 - \Xi(\sigma)^2}$

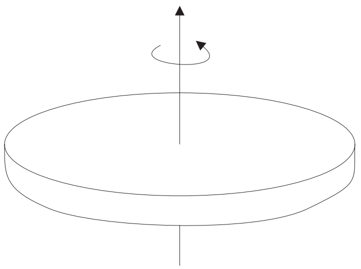
we see that $r_0 = 0$ at $r=1/\Omega \Rightarrow$ Blackfolds ends here

$\Rightarrow$ The blackfold $\mathcal{B}_2 = D_2$: A rotating disc

This is actually the ultraspinning limit of a MP black hole! (with one ang. momentum)

Evidence: The MP BH precisely becomes pancake shaped

Also: The r_0 as function of r the same, and M, J and A_H the same (once we identify κ and Ω)



Blackfolds describe the ultraspinning limits of MP black holes!

Can be readily generalized to several ultraspins (filled ellipsoids replace the disc)

Black bestiary (compendium of beasts):

Blackfold construction shows the existence of new types of asymptotically flat black holes in higher dimensions

Hole → S^2 (4D)

Ring → $S^1 \times S^2$ (5D)

2-torus → $T^2 \times S^2$ (6D)

3-torus → $T^3 \times S^2$ (7D)

3-sphere → $S^3 \times S^2$ (7D)

$S^1 \times S^3$ → $S^1 \times S^3 \times S^2$ (8D)

4-torus → $T^4 \times S^3$ (9D)

5-sphere → $S^5 \times S^2$ (9D)

$T^2 \times S^3$ → $T^2 \times S^3 \times S^2$ (9D)

4D: S^2

5D: S^3, $S^1 \times S^2$

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8D: S^6, $S^1 \times S^5$, $T^2 \times S^4$, $T^3 \times S^3$, $S^3 \times S^3$, $S^1 \times S^3 \times S^2$

9D: S^7, $S^1 \times S^6$, $T^2 \times S^5$, $T^3 \times S^4$, $S^3 \times S^4$, $S^1 \times S^3 \times S^3$, $T^4 \times S^3$, $S^5 \times S^2$, $T^2 \times S^3 \times S^2$

We write here all the objects as $\mathcal{B}_p \times S^{D-p-2}$

For each of the MP black holes we have ultraspinning limits with filled ellipsoids

<u>Lesson from Blackfold approach</u>:

Product-topology: $A \times B$

For example: $S^3 \times S^3$

Supported by mechanical equilibrium (arrow to the first S^3)

Supported by internal structure of BH, like the S^2 of the 4D Schwarzschild BH (arrow to the second S^3)

In a purely topological analysis: One cannot distinguish how the manifolds are supported.

An important ingredient in classifying black holes

Caveats

- 1st order correction to metric: Regularity of black brane horizon after bending ?
  - shown for black rings
  - extension to general odd-spheres
    (use matched asymptotic expansion)
  - We know that it works for Evenballs (filled ellipsoids)

- 2nd order correction to metric: backreaction of blackfold on background geometry
  - Important, but we have yet to do this:
    could make it impossible for some leading-order solutions to remain stationary
    (must be analyzed case-by-case)
  - We know that it works for Evenballs (filled ellipsoids)

- Stability of blackfolds
  - Blackfolds are unstable to the Gregory-Laflamme instability since
    the horizon scale of the p-brane is much smaller than the blackfold scale
    $\Rightarrow$ Instability under short wavelength perturbations ($\lambda \sim r_0$)
  - We can use our blackfold equations to analyze stability under long wavelength
    perturbations ($\lambda \sim R \gg r_0$)

Other cases

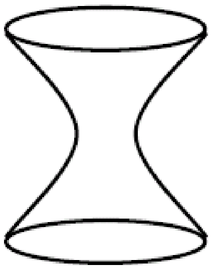
► static minimal blackfolds $\tau_{ij} = -P\eta_{ij}$

(no boost)

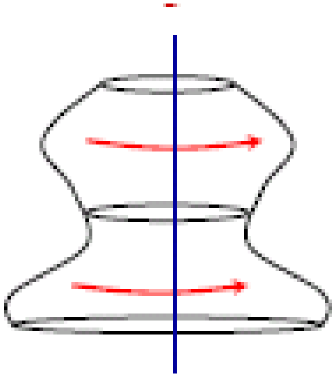
$\longrightarrow$ $K^\rho = 0$ (mean curvature vector)

minimal submanifold

e,g. hyperboloid (static non-compact blackfold)



► axisymmetric blackfolds



use numerics or further perturbative approach ?

New blackfolds in 5D: helical rings and strings

Emparan,TH,Niarchos,Obers (in progress)

for black 1-folds we can take curves with tangent vector equal to
a linear combination of isometries $\quad \zeta = \sum_i c_i \xi^{(i)} |_{x = X(\sigma)}$

$\rightarrow$ for critical boost this satisfies Carter + blackness

- helical black string: $\zeta \sim (k\partial_x + \partial_\phi)|_{r = R}$

  - helix with pitch k
  - boost along string gives momentum along x and angular momentum along ϕ

- helical black ring: $\zeta \sim (n\partial_\phi + m\partial_\psi)|_{r_1 = R_1, r_2 = R_2}$

  - helix of radius R_2 around circular trajectory of radius R_1 that closes on itself after m turns
  - boost is linear combo of two angular momenta

has only single spatial U(1) isometry:
first evidence of such a solution in 5D ! (as admited by rigidity theorem)

Hollands, Ishibashi, Wald

Charged blackfolds:

We have seen that we can wrap black p-branes on certain compact submanifolds and spin them up, so that they are in both thermodynamic and mechanic equilibrium

Can we do the same for charged black p-branes?
E.g. black Dp-branes in String Theory?

Wrap black D-string on circle and spin it up $\Rightarrow$ Dipole black ring
Exact dipole black ring solution in 5D   Emparan

Can we do this more generally?
Can we wrap D-branes on odd-spheres?

We have seen that blackfolds in pure gravity are unstable due to the Gregory-Laflamme instability
$\Rightarrow$ This instability can be avoided for charged blackfolds
since thin charged black p-branes can be stable for sufficiently large charge

Charged blackfold equations:

$$K_{\alpha\beta}^{\rho}\tau^{\alpha\beta} = F^{\rho\nu_1\cdots\nu_{p+1}}J_{\nu_1\cdots\nu_{p+1}} \qquad \nabla_\mu J^{\mu\nu_1\cdots\nu_{p+1}} = 0$$

Carter equation in external field F — Charge conservation

Example: Wrapping of a black D-brane on an odd-sphere S^p (p odd)

Locally boosted black Dp-brane:

r_0: Horizon radius

α,ν_i: Boost parameters

η: Charge parameter

We embed the brane in Euclidean space with no external field: F = 0

Local charge of D-brane: $$Q = \int_{S^{n+1}} *F$$

We impose blackness: κ and Ω_i constant
as well as constant charge Q

Result: $\Rightarrow$ We have wrapped a black D-brane on a sphere S^p, p odd

Give rise to new solutions in String Theory for p > 1. Can play important role

New higher types of Dipole charges

Summary of first part:

- ▶ D > 4 BHs is a much richer and more complicated subject than D = 4 BHs
- ▶ Myers-Perry BH not the only solution
- ▶ Intuition: Take a black string, bend it in a circle and spin it up

  $\Rightarrow$ Gives a black ring!

This intuition works: Exact black ring solution in 5D

Approximate solution for D $\geq$ 6

- ▶ Reason for richness for D > 4 BHs: We can have widely separated scales

This occurs in the regime: $J \gg M(GM)^{\frac{1}{D-3}}$ with $J = (\sum_i J_i^2)^{1/2}$

- ▶ More general framework: **Blackfolds**

Action for blackfolds: $I_{\text{brane}} \propto \int \sqrt{h}\left(1 - \Xi(\sigma)^2\right)^{\frac{n}{2}}$

Examples:

- Odd-spheres, and products thereof
- Even-balls = Ultraspinning limit of Myers-Perry BHs

New approaches to higher-dimensional black holes

- Blackfolds and Domain Structure

Troels Harmark

Niels Bohr Institute

IST String Fest at Instituto Superior Tecnico
Lisbon, Portugal, June 29 – July 1, 2009

Based on
arXiv:0904.4246 "Domain Structure of Black Hole Space-Times"
PRL 102: 191301 (2009) with Emparan, Niarchos and Obers
JHEP 0170:110 (2007) with Emparan, Niarchos, Obers and Rodriguez
and other less recent papers.

Outline of talk:

Recap of last time

Review of rod-structure and 5D solutions

Domain structure

Conclusions

Recap of last time – The blackfold approach:

$L_J \lesssim L_M$: BHs behave qualitatively like in 4D

$L_J \sim L_M$: Threshold of new dynamics

$L_J \gg L_M$: Blackfold regime

$$L_M = (GM)^{1/(D-3)}$$

$$L_J = \frac{1}{M}\sqrt{\sum_i J_i^2}$$

Procedure for making a blackfold: Take a probe black p-brane, wrap it on a submanifold $\mathcal{B}_p$ and spin it up

We consider embedding $X^\mu(\sigma)$ of (p+1)-dimensional submanifold $\mathbb{R} \times \mathcal{B}_p$ in D-dim. Minkowski space $\mathbb{R} \times \mathbb{R}^{D-1}$

Blackfold equations:

$$(\nabla_\alpha^{(h)} \partial_\beta X^\rho + \Gamma^\rho_{\mu\nu} \partial_\alpha X^\mu \partial_\beta X^\nu)\tau^{\alpha\beta} = 0$$

Equation for mechanical equilibrium of blackfold

$\tau^{\alpha\beta}$: Locally given by boosted (&rotated) EM tensor for static flat black p-brane

Using blackness condition: Determined fully from $X^\mu(\sigma)$ and Ω_i

Blackness condition $\Leftrightarrow$ Thermodynamical equilibrium

Action principle:

$$I_{\text{brane}} \propto \int \sqrt{h}\left(1-\Xi(\sigma)^2\right)^{\frac{n}{2}}$$

where h is the minus determinant of $h_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X^\nu g_{\mu\nu}$

and $\Xi(\sigma) \equiv \sqrt{\sum_{i=1}^{m} (r_i(\sigma))^2 \Omega_i^2}$ is the velocity field

Gives very powerful method to derive equations for Blackfolds!

Geometric censorship: We can censor embedding geometries

Examples:

- ► Odd-spheres: Embedding in $\mathbb{R}^{D-1}$: Round S^p, p odd

  $I_{\text{brane}} \propto \int R^p (1-\Omega^2 R^2)^{\frac{n}{2}}$ EOM solved by $R = \sqrt{\frac{p}{n+p}}\frac{1}{\Omega}$

  Also products of odd-spheres

- ► Rotating disc: Rigidly rotating disc, ends at r = 1/Ω

  Thickness: $r_0(\sigma) = \frac{n}{2\kappa}\sqrt{1-\Xi(\sigma)^2}$ with Ξ = r Ω

  Ultraspinning limit of Myers-Perry

  Similar for several ultraspins: Evenballs a.k.a. filled ellipsoids

Black bestiary (compendium of beasts):

Blackfold construction shows the existence of new types of asymptotically flat black holes in higher dimensions

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We write here all the objects as $\mathcal{B}_p \times S^{D-p-2}$

For each of the MP black holes we have ultraspinning limits with filled ellipsoids

Blackfold approach succesful in understanding higher-dimensional BHs in the blackfold regime $L_J \gg L_M$

But we still lack a good understanding of the "space of BH's for D>4" in general

- ▶ How to characterize a BH space-time?
- ▶ What is the minimal information you need to provide?
- ▶ We've already seen that asymptotic "charges" M, J_i, etc. are not enough for D > 4

⇒ This is the main subject of this second part of my talk

Symmetries of black holes
5D black holes and the rod-structure
Domain structure of black hole space-times

Symmetries of black holes:

Killing vector fields $\leftrightarrow$ Symmetries of the black hole space-time

Stationary black holes $\Rightarrow$ Exists an asymptotically time-like Killing vector field

D - # of commuting Killing vector fields = # of coordinates that metric depends on

Symmetries of 4D black holes:

Rigidity theorem (Hawking):

A stationary black hole has an axial Killing vector field

$\Rightarrow$ All 4D black holes have two commuting Killing vector fields

Important part of 4D black hole uniqueness theorems
Weakness: Assumes analyticity

Two commuting Killing vector fields: The maximal number possible
for an asymptotically flat 4D BH space-time
(the BH breaks translational invariance of the Poincare group)

Symmetries of D>4 black holes:

Rigidity theorem (Hollands, Ishibashi & Wald; Moncrief & Isenberg)

Consider a stationary black hole (not static)

Exists a (non-time-like) Killing vector field generating a U(1)

$\Rightarrow$ Corresponds to rotation in a rotation plane

All D > 4 black holes have two commuting Killing vector fields

However, the maximal number of commuting Killing vector fields for a D>4 black hole space-time is

$$1 + [(D-1)/2]$$

Stationarity (→ 1) ; # of rotation planes (→ [(D-1)/2])

D	1 + [(D-1)/2]	Difference
4	2	2
5	3	2
6	3	3
7	4	3
8	4	4
9	5	4

As D gets higher, the metric depends on more coordinates
$\Rightarrow$ Harder to find exact solutions!

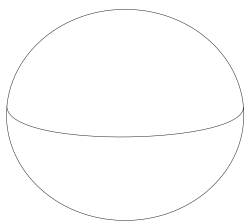
Also: We cannot exclude black holes with less than the maximal number of commuting Killing vector fields

Review of 5D black holes and the rod-structure:

We already mentioned the two exact solutions:

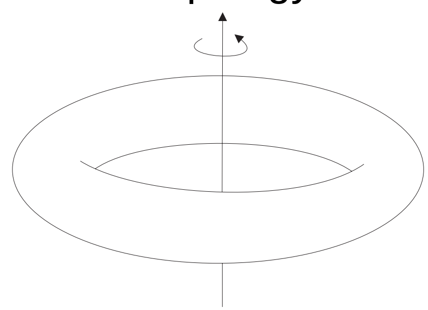
Myers-Perry black hole

Horizon topology S^3



Black ring of Emparan-Reall

Horizon topology $S^1 \times S^2$



Both of these solutions have 2 rotational Killing vector fields, hence they have 3 commuting Killing vector fields

What other possibilities are there for D=5?

For D=5 we have two rotation planes

Consider stationary solutions which are rotationally symmetric in both rotation planes

$\Rightarrow$ We assume three commuting Killing vector fields generating $\mathbb{R} \times U(1)^2$

Given this, we can write the metric on the canonical form

$$ds^2 = G_{ij} dx^i dx^j + e^{2\nu}(dr^2 + dz^2) \ , \ r = \sqrt{|\det G_{ij}|}$$

TH (Generalizing Weyl, Papapetrou, Emparan & Reall)

G(r,z): A 3 x 3 matrix field being the metric on the Killing directions

We now have the very important **decoupling property** of the Einstein equations:

Equations for $G_{ij}(r,z)$ decouples from the $\nu(r,z)$ function:

$$\left(\partial_r^2 + \frac{1}{r}\partial_r + \partial_z^2\right) G = \partial_r G G^{-1} \partial_r G + \partial_z G G^{-1} \partial_z G$$

Given any $G_{ij}(r,z)$ solution we can integrate equations for $\nu(r,z)$

$\Rightarrow$ We only have to solve the equations for $G_{ij}(r,z)$

Moreover: The equations for $G_{ij}(r,z)$ are integrable! Pomeransky

This leads to powerful solution generating techniques

What is the space of solutions that one can generate?

This is parameterized by the rod-structure TH

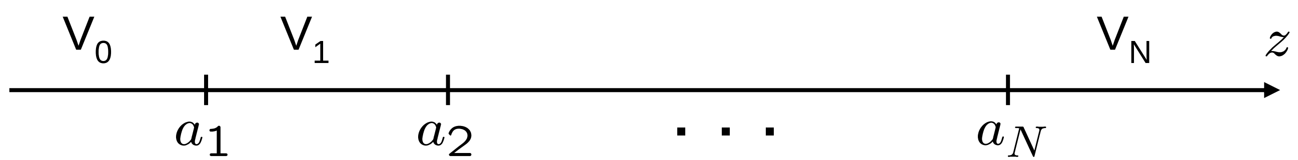
Rod-structure: The structure of the fixed points of the Killing vector fields

A fixed point: The Killing vector flow stays in the same point

Typically either: 1) Fixed point of a rotation, or 2) An event horizon

For a fixed point det(G) = 0 hence r=0
$\Rightarrow$ The rod-structure resides on the z-axis

z-axis divided into intervals depending on which Killing vector field has a fixed point:



Intervals $[a_i,a_{i+1}]$: Called rods Killing vector field V_i: Direction of rod $[a_i,a_{i+1}]$

The rod-structure provides invariants of the black hole space-time

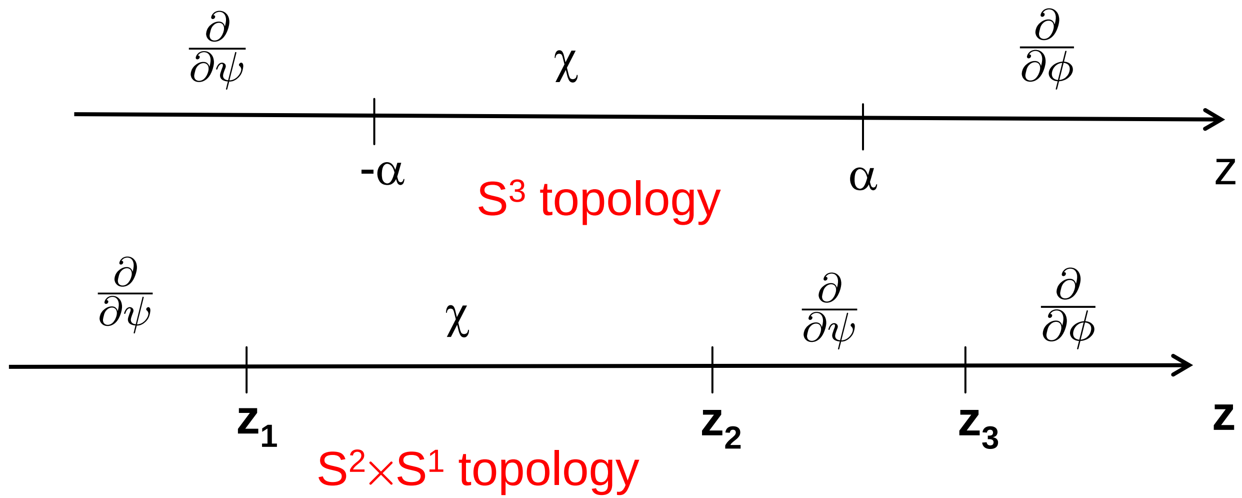
Examples of D=5 asymptotically flat solutions:

In the following: χ a Killing vector field corresponding to a Killing horizon (event horizon)

$\chi = \frac{\partial}{\partial t} + \Omega_1 \frac{\partial}{\partial \phi} + \Omega_2 \frac{\partial}{\partial \psi}$ $\quad$ **Ω_1 , Ω_2 :** angular velocities

Myers-Perry black hole:

Black ring:



One can read off the topology from the rod-structure

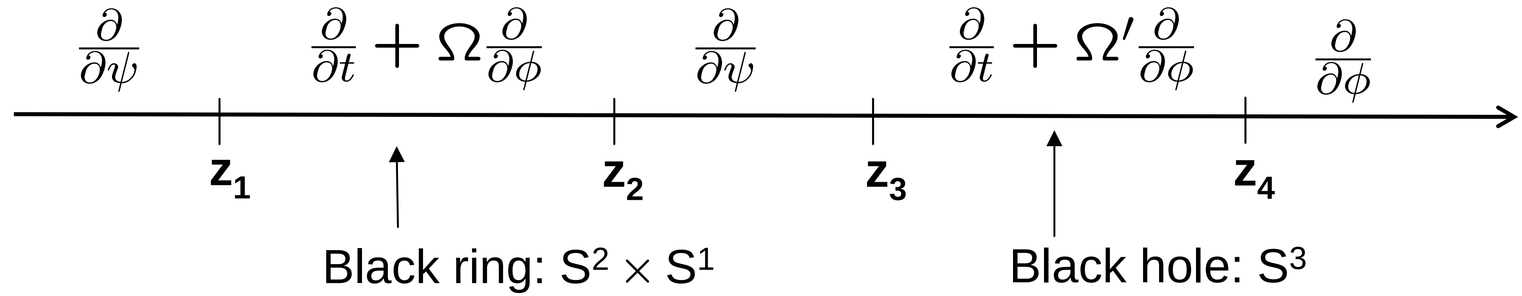
For event-horizon rods: Two circles are fibered over an interval

S^3 topology: Two different circles shrink to zero at the two endpoints

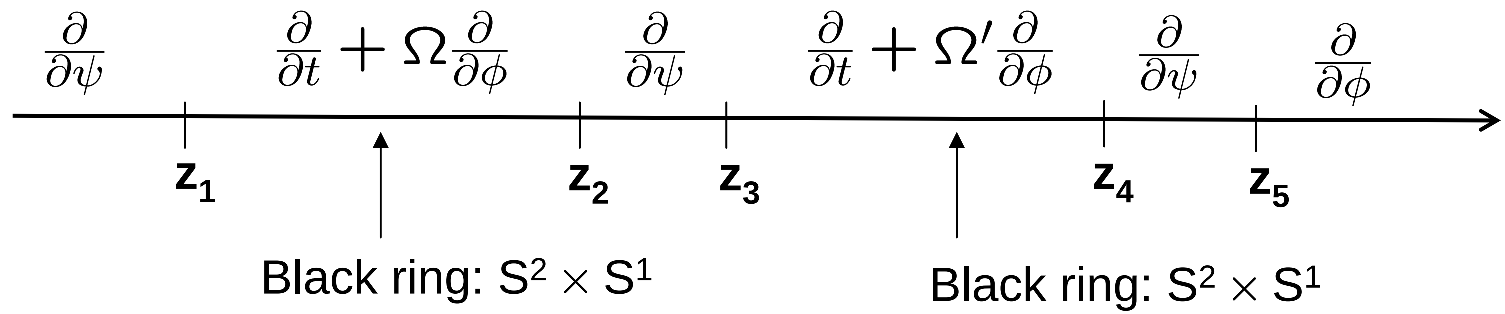
$S^2 \times S^1$ topology: The same circle shrinks to zero at the two endpoints, the other circle stays finite

Regular metrics found in D=5 with disconnected horizon topology:

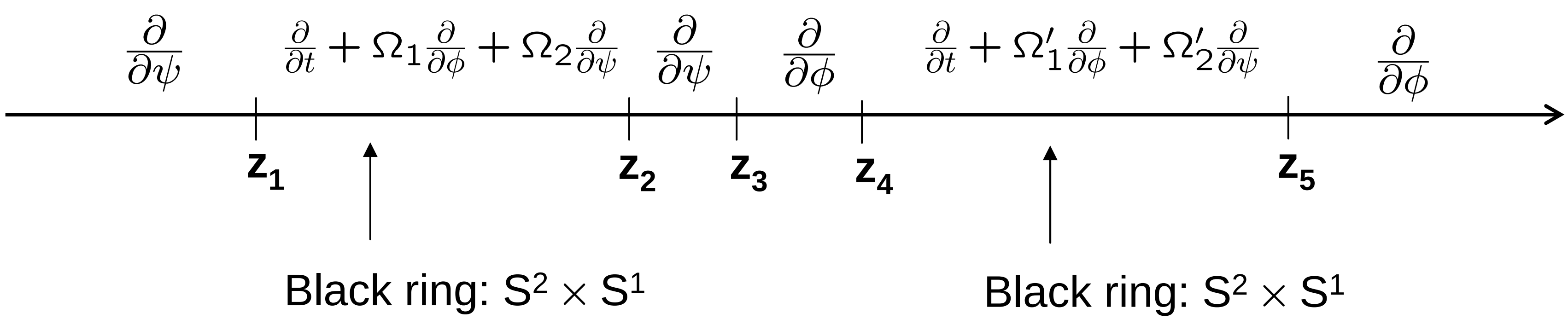
Black Saturn (Elvang & Figueras):



Di-ring (Iguchi & Mishima):



Bicycling black rings (Izumi;Elvang & Rodriguez):

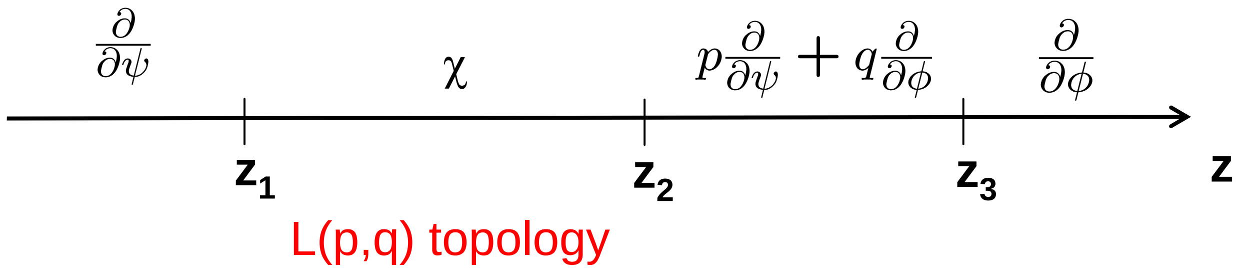


Irregular solutions found:

(widening of parameter space could give regular solutions)

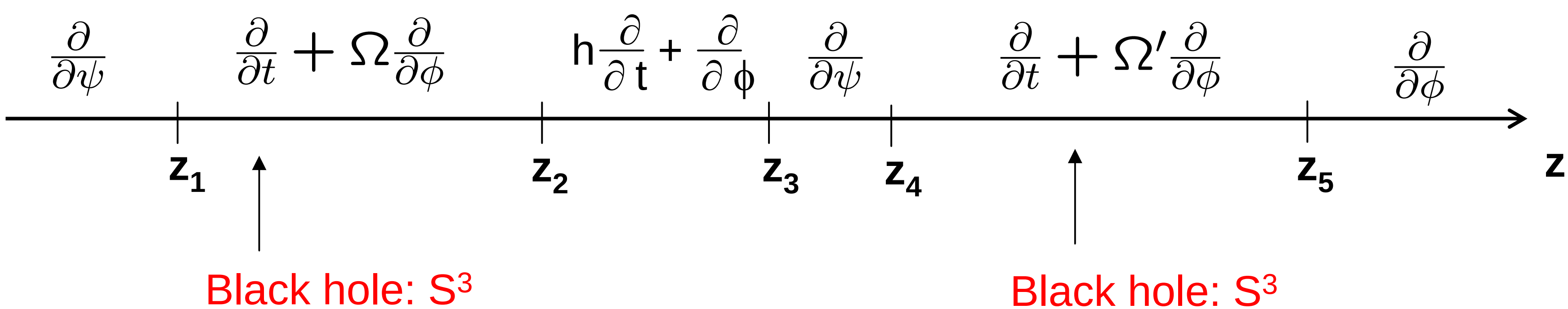
Black lens:

Hollands & Yazadjiev. Evslin. Chen & Teo.



Regularity on space-like rod requires p,q to be integers

A double Myers-Perry black hole (Herdeiro, Rebolo, Zilhao & Costa):



Status on uniqueness theorems for D=5:

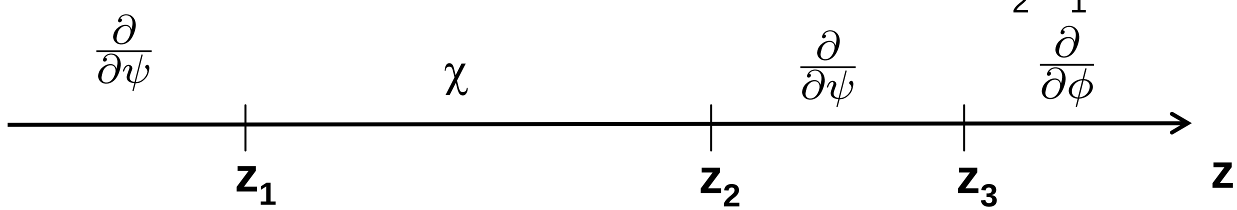
We have already seen that you do not have uniqueness given M, J_1 and J_2

Conjecture: The rod-structure gives a classification of all
D=5 black hole space-times with 2 rotational Killing vector fields TH

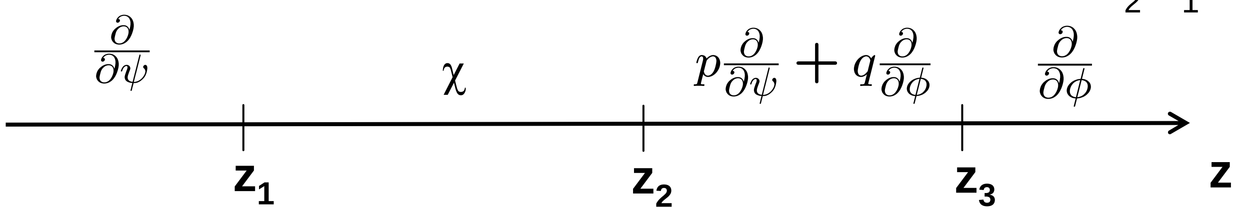
What has been proven so far:

Uniqueness of Myers-Perry BH given M, J_1 and J_2 Ida & Morisawa

Uniqueness of Black ring given M, J_1, J_2, and $\frac{z_3-z_2}{z_2-z_1}$ Hollands & Yazadjiev



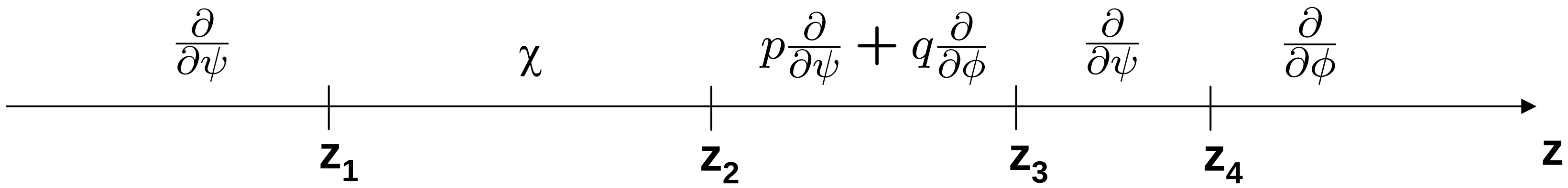
Uniqueness of Black lens given M, J_1, J_2, p, q, and $\frac{z_3-z_2}{z_2-z_1}$ Hollands & Yazadjiev



(assumes existence of regular Black lens)

So far: Uniqueness proven for all single-horizon BH's given the rod-structure
and the asymptotic charges M, J_1 and J_2

Note: Uniqueness theorem of Hollands and Yazadjiev can trivially be extended to any rod-structure configuration with a single horizon, e.g. to the configuration:



Caveat:

► Still no general uniqueness theorem for all known 5D black hole space-times, i.e. with more than one horizon

$\Rightarrow$ Still work to be done on 5D uniqueness given 2 rotational Killing vector fields

Domain structure of black hole space-times:

So far, the rod-structure in 5D have been our best bet in characterizing the rich space of higher-dimensional asymptotically flat black holes

Limitations of the rod-structure:

- It cannot work for D $\geq$ 6 asymptotically black hole space-times
  Reason: Necessary to have D-2 commuting Killing vector fields to define the rod-structure
- Its definition relies on using the Einstein equations everywhere in the space-time $\Rightarrow$ Not generic, depends on details of the theory
- We cannot use it for 5D BHs with only one rotational Killing vector field
  Our blackfold construction of a helical black ring suggests that there are 5D black hole space-times with only one rotational Killing vector field
- No definition for AdS or dS black holes in any dimension

In the following we shall attack all these limitations

Consider D-dim. space-time (Lorentzian manifold) $\mathcal{M}_D$

Assume $\mathcal{M}_D$ has p commuting Killing vector fields $V_{(i)}$, i = 0,1,...,p-1

We assume $V_{(0)}$ generates $\mathbb{R}$ while $V_{(i)}$, i=1,2,...,p-1, generates U(1)

<u>Basic idea</u>: Consider structure of fixed points of $V_{(i)}$

First thing to do: Find canonical form for metric

We can always find coord's $x^0,\ldots,x^{p-1},y^1,\ldots,y^n$ such that

$$V_{(i)} = \frac{\partial}{\partial x^i}$$

n = D - p

Metric: $$ds^2 = G_{ij}(dx^i + A^i)(dx^j + A^j) + \tilde{g}_{ab}dy^a dy^b$$

$$A^i = A^i_a dy^a$$

G_{ij}: Metric on Killing vector space

Fixed point of Killing vector field W ⇔ zero distance when going along W at fixed point ⇔ norm of W = 0 at fixed point ⇒ det(G_{ij}) = 0 at fixed point

⇒ We define $r(y) = \sqrt{|detG_{ij}|}$

We see that fixed points are located at r = 0

Define the n-dim. manifold $\mathcal{N}_n$ from $\mathcal{M}_D$ by modding out points connected by integral curves of a linear combo of $V_{(i)}$

Natural metric on $\mathcal{N}_n$: $ds_n^2 = \tilde{g}_{ab} dy^a dy^b$

On $\mathcal{N}_n$ the one-forms $A^i = A^i_a dy^a$ can be thought of as gauge fields

Assume $\left(\frac{\partial r}{\partial y^1}, \cdots, \frac{\partial r}{\partial y^n} \right) \neq 0$ almost everywhere in $\mathcal{N}_n$

General argument (not relying on Einsteins equations) shows one can find z^α, $\alpha = 1,\ldots,n-1$ such that $(r, z^1, \ldots, z^{n-1})$ is a coordinate system on $\mathcal{N}_n$ and the metric has $g_{rz^\alpha} = 0$

⇒ We can write the metric of $\mathcal{N}_n$ on the form

$$ds_n^2 = e^{2(n-1)\nu} dr^2 + e^{2\nu} \Lambda_{\alpha\beta} dz^\alpha dz^\beta$$

Define $\lambda = \sqrt{|\det(\Lambda_{\alpha\beta})|}$

One can show it is possible to choose $\lambda = 1$ for D-dim. Minkowski space $\mathbb{R}^{1,D-1}$ (and also for Kaluza-Klein space $\mathbb{R}^{1,D-1-q} \times T^q$)

⇒ We can demand that $\lambda \to 1$ for $r \to \infty$ for asymptotically flat space-times (and asymptotically KK space-times)

Canonical form of metric for asymptotically flat (or KK-space) space-times:

The canonical form for asymptotically flat space-times $\mathcal{M}_D$ (or asymptotically KK-space):

$$ds^2 = G_{ij}(dx^i + A^i)(dx^j + A^j) + e^{2(n-1)\nu} dr^2 + e^{2\nu} \Lambda_{\alpha\beta} dz^\alpha dz^\beta$$

$$r = \sqrt{|detG_{ij}|} \qquad \lambda \to 1 \text{ for } r \to \infty$$

(λ is defined as $\lambda = \sqrt{|\mathrm{det}(\Lambda_{\alpha\beta})|}$)

Notice: We haven't used Einstein equations to derive this canonical form!

→ It works irrespective of what matter fields is put in the theory
E.g. for charged black holes, charged dilatonic black holes, non-extremal D-branes, etc., etc.

What coordinate transformations preserve this form?

- Rigid rotations of x^i
- "Gauge transformations" $x^i \to x^i - \alpha^i(r, z^\alpha)$
- (n-1)-Volume preserving diffeomorphisms of $(z^1, \ldots, z^{n-1})$

What about asymptotically Anti-de Sitter space?

General argument still works, so we can write the canonical form

$$ds^2 = G_{ij}(dx^i + A^i)(dx^j + A^j) + e^{2(n-1)\nu} dr^2 + e^{2\nu} \Lambda_{\alpha\beta} dz^\alpha dz^\beta$$

$$r = \sqrt{|detG_{ij}|} \qquad \lambda \to \lambda_0 \text{ for } r \to \infty$$

Where $\lambda_0(r,z^\alpha)$ is defined from Anti-de Sitter space.

Still some work to do... de-Sitter space is more tricky because of cosm. horizon

Now that we found the canonical form of the metric we are ready for

Defining the domains and their directions:

Define: $B = \{q \in \mathcal{N}_n | \det G(q) = 0\}$

$\Rightarrow$ B is the set of fixed points of the Killing vector fields

We now want to study the structure of the set B

A natural metric on B: $ds^2 = \Lambda_{\alpha\beta}|_{r=0} dz^\alpha dz^\beta$

Define the sets $Q_k = \{q \in \mathcal{N}_n | \dim(\ker G(q)) \geq k\}$

We see immediately that $Q_0 = \mathcal{N}_n$ and $Q_1 = B$

Theorem: Consider a point $q \in Q_{k+1} - Q_{k+2}$
Then Q_{k+1} is a codimension 1 submanifold of Q_k in a neighborhood of q

⇒ The sets Q_k introduces a hierarchy of submanifolds of $\mathcal{N}_n$

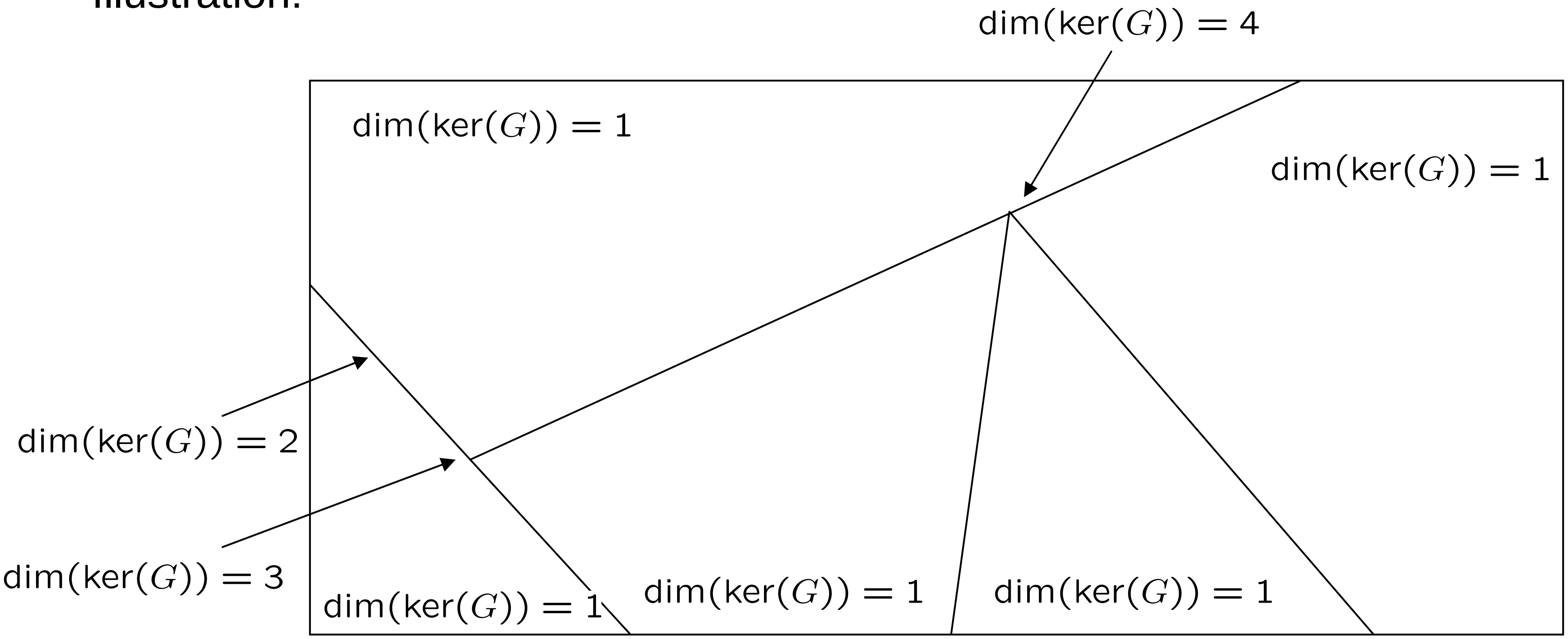
Theorem: The vector space ker(G) is constant in connected pieces of B – Q_2

with this we can define domains:

Definition: Let q ∈ B – Q_2 and W ∈ ker G(q)

A **domain** D containing q is the maximal connected set in B such that q ∈ D and such that for any point q' ∈ D we have W ∈ ker G(q')

Illustration:



Theorem: Let $D_1,\ldots,D_N$ be the domains of B. Then we have:

- $B = D_1 \cup \cdots \cup D_N$
- For each domain D_m we can find a Killing vector field W_m such that $W_m \in \ker(G)$ for all points in D_m

  We call W_m the **direction** of the domain D_m

- If W_m is space-like for $r \to 0$ we can write it in the form

$$W_m = \sum_{i=1}^{p-1} q_i V_{(i)}$$

  where q_i are integers and relatively prime numbers
  In this case we say the direction W_m is space-like

- If W_m is time-like for $r \to 0$ we can write it in the form

$$W_m = V_{(0)} + \sum_{i=1}^{p-1} \Omega_i V_{(i)}$$

  and the domain D_m is a Killing horizon for the Killing vector field W_m
  In this case we say the direction W_m is time-like

Definition: The domain structure of a solution is defined as the split-up of B in domains $D_1,...,D_N$ up to (n-1)-volume preserving diffeomorphisms, along with the directions W_m, m=1,2,...,N of the domains

The above results mean that the domain structure gives invariants of the black hole space-time that we consider

The invariants include the topological structure of the split-up in domains, the directions of the domains, as well as the (n-1)-dimensional volumes of the domains measured wrt. the metric

$$ds^2 = \Lambda_{\alpha\beta}|_{r=0} dz^\alpha dz^\beta$$

D=5 asymptotically flat black hole space-times:

Assume 2 rotational Killing vector fields (D=5, p=3, n=2)

Canonical form of metric:

$$ds^2 = G_{ij}(dx^i + A^i)(dx^j + A^j) + e^{2\nu}(dr^2 + \lambda^2 dz^2)$$

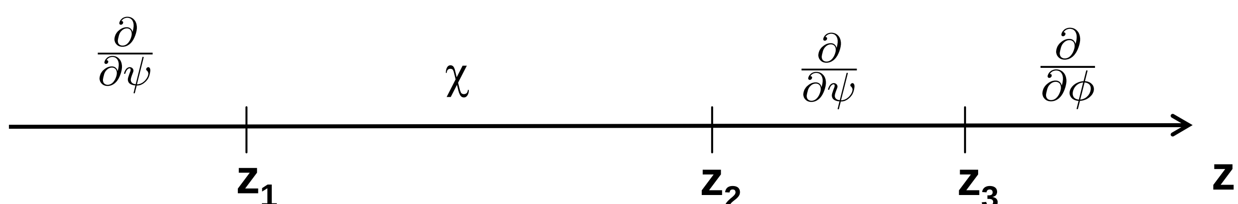
$$r = \sqrt{|detG_{ij}|} \qquad \lambda \to 1 \text{ for } r \to \infty$$

Provides general definition of rod-structure for all asymptotically flat 5D BH space-times

Rod-structure previously only defined for pure gravity and particular case of Einstein-Maxwell gravity (using Einstein equations) TH. Hollands and Yazadjiev

$$Q_0 = \{(r,z)|r \geq 0\},\ Q_1 = \{(r,z)|r = 0\},\ Q_2 = \text{rod endpoints}$$

Black ring:



$S^2 \times S^1$ topology

D=6 asymptotically flat black hole space-times:

Assume 2 rotational Killing vector fields

$$W_1 = \frac{\partial}{\partial \phi_1} \qquad W_2 = \frac{\partial}{\partial \phi_2}$$

D=6, p=3, n=3

$$Q_0 = \mathcal{N}_3 = \{(r, z^1, z^2) | r \geq 0\}$$

$$Q_1 = B = \{(z^1, z^2) \in R^2\}$$

Domain structure lives on a plane!

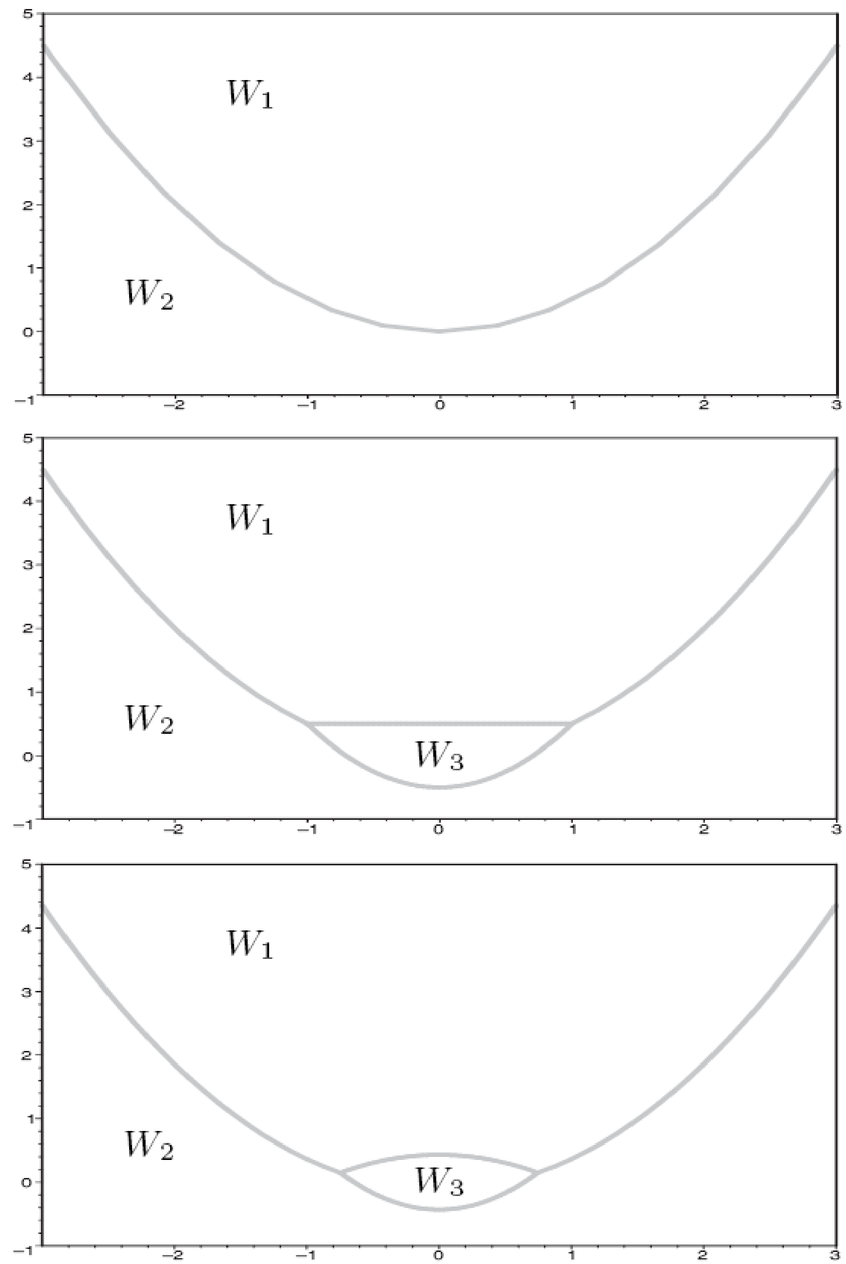
Domain structure of known exact solutions

- Minkowski space
- Schwarzschild-Tangherlini

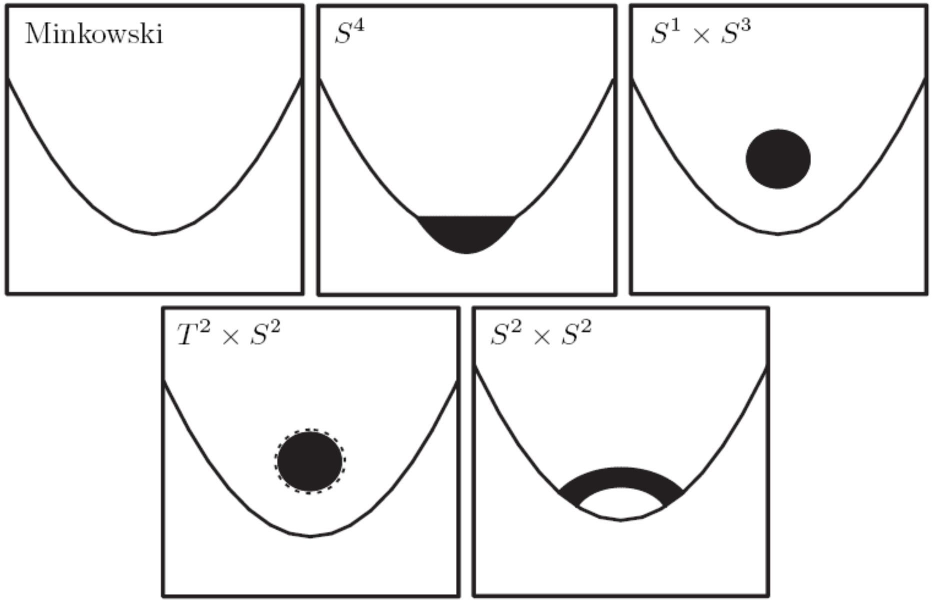
$$W_3 = \frac{\partial}{\partial t}$$

- Myers-Perry

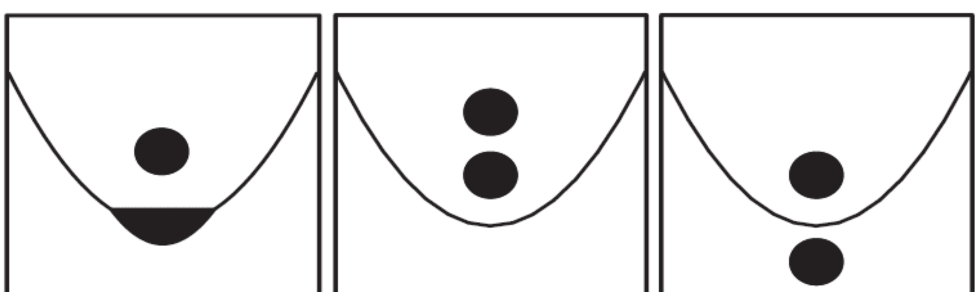
$$W_3 = \frac{\partial}{\partial t} + \sum_{i=1}^{2} \Omega_i \frac{\partial}{\partial \phi_i}$$



More general 6D domain structures:



Multiple horizons:



D=7 asymptotically flat black hole space-times:

Assume 3 rotational Killing vector fields

$$W_1 = \frac{\partial}{\partial \phi_1} \qquad W_2 = \frac{\partial}{\partial \phi_2} \qquad W_3 = \frac{\partial}{\partial \phi_3}$$

D=7, p=4, n=3

$$Q_0 = \mathcal{N}_3 = \{(r, z^1, z^2) | r \geq 0\}$$

$$Q_1 = B = \{(z^1, z^2) \in R^2\}$$

Domain structure lives on a plane!

Domain structure of known exact solutions

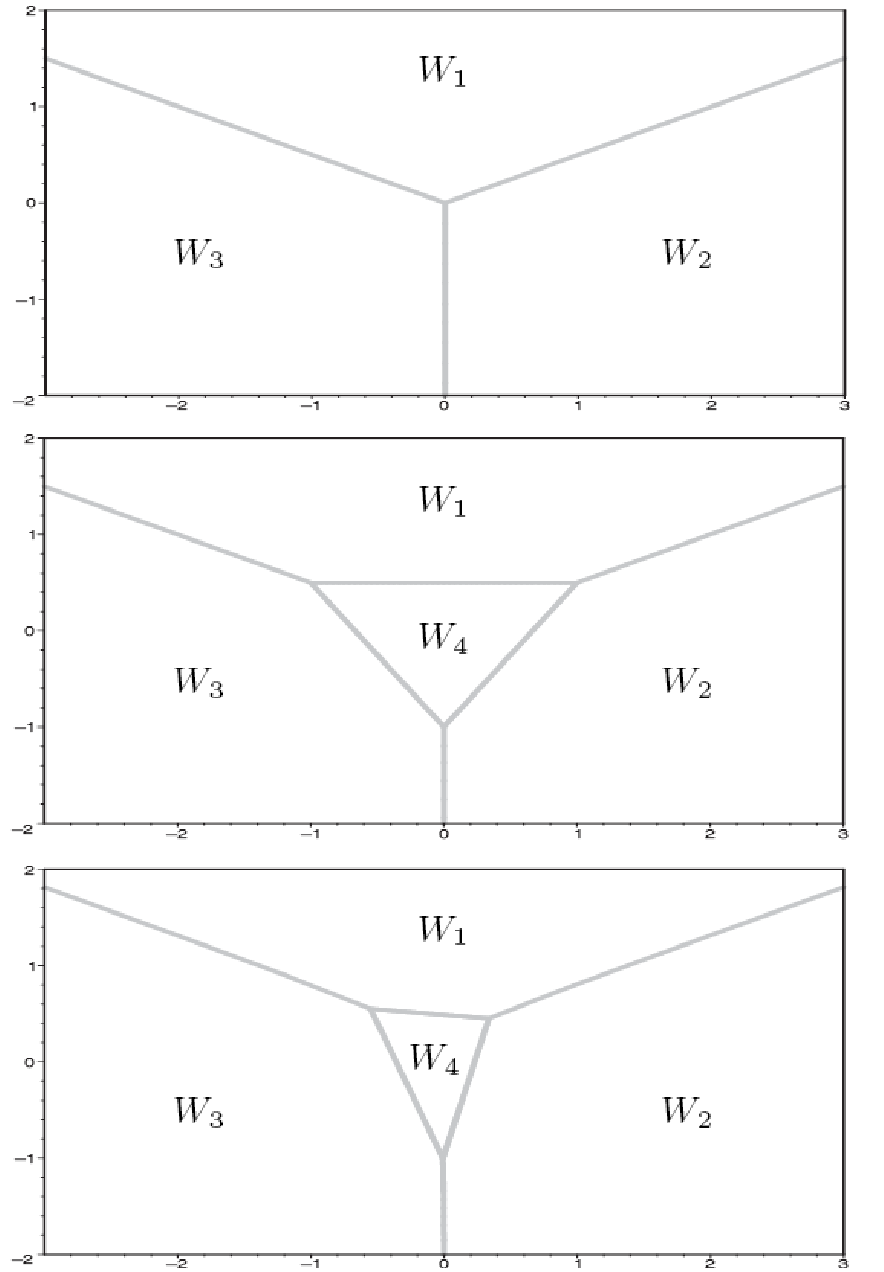
- Minkowski space

- Schwarzschild-Tangherlini

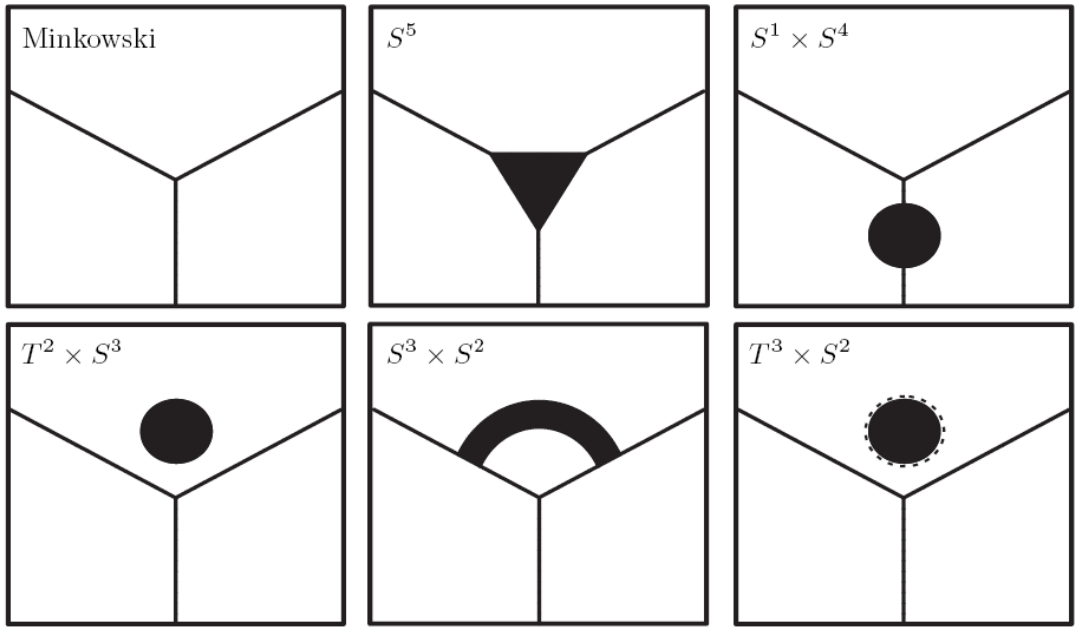
$$W_4 = \frac{\partial}{\partial t}$$

- Myers-Perry

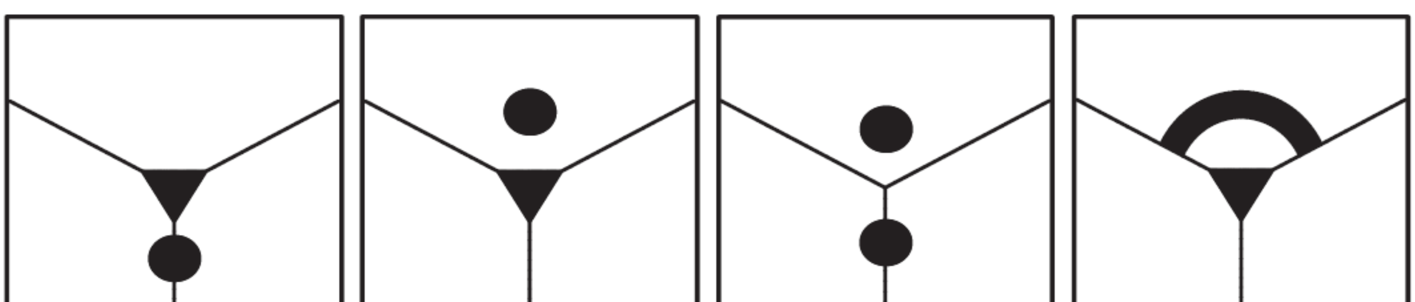
$$W_4 = \frac{\partial}{\partial t} + \sum_{i=1}^{3} \Omega_i \frac{\partial}{\partial \phi_i}$$



More general 7D domain structures:



Multiple horizons:



General remarks on domain structure:

- The domain structure provides invariants of black hole space-times, both topological and continuous

- The more Killing vector fields we have, the more powerful the domain structure classification becomes

- The domain structure always contain the information about the horizon topology, no matter how many Killing vector field there are

- For maximal number of Killing vector fields:
  Domain structure (plus asymptotic charges) enough to fully characterize a solution)?

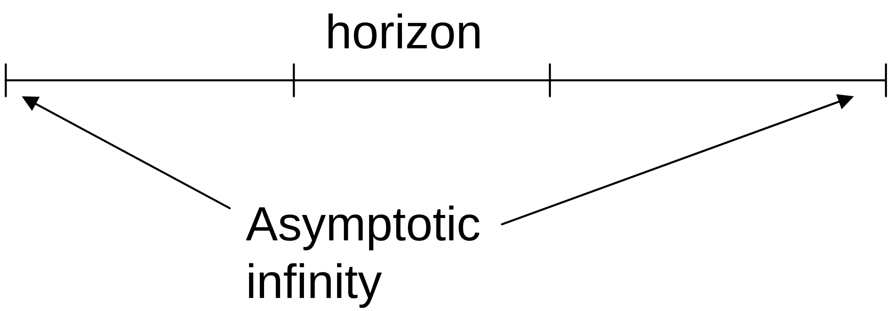
  Seems possible for pure gravity black hole space-times

  In theories with electric/magnetic matter:
  Need to specify the dipole charges (in addition to the total charges)

(work in progress with Armas and Caputa)

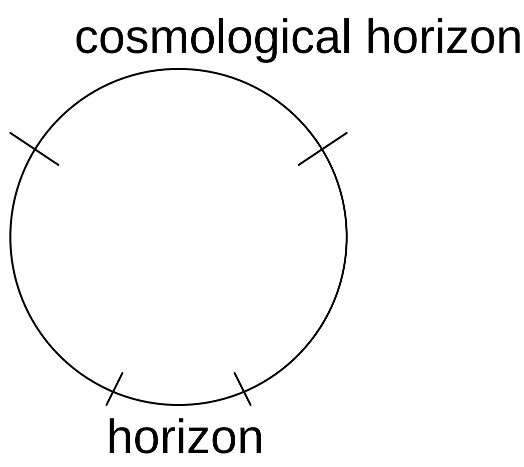
- For D=4,5 AdS black holes (assuming n=2 for D=5)

  Rod-structure is compact:



- For D=4,5 dS black holes (assuming n=2 for D=5)

  Rod-structure is periodic:



Conclusion:

4D: Things are simple, only one BH phase available given the asympt. charges

D>4: $L_J \gg L_M$ possible $\Rightarrow$ Blackfolds $\Rightarrow$ Many new types of BH's

$L_J \ll L_M$ versus $L_J \gg L_M$ regimes $\Rightarrow$ Complicated problem

D>4 BH's a rich subject

How to distinguish BH's?
No longer possible with asymptotic charges M, J_i, etc.

We need a new set of invariants of the BH space-time

The domain structure provides such a set of new invariants

A full classification? Could be for pure gravity
With matter: We need to add dipole charge

D=5: We made significant progress on understand "the space of BH's"
D>5: We only just started, much work to be done