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AdS/CFT in the Veneziano Limit

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with A. Gadde and E. Pomoni

11 years of AdS/CFT

Paradigm: $\mathcal{N} = 4$ SYM $\leftrightarrow$ IIB on $AdS_5 \times S^5$ Maldacena

Extremely rich example

All other gravity duals of 4d gauge theories are rather close cousins of this case: motivated from $D3$ branes at local singularities in critical string theory

• Adjoint or bifundamental matter (quivers).
Fundamental flavors can be added in probe approximation $N_f \ll N_c$

• Susy can be broken but there are always remnants of the "extra" matter

• Anomaly coefficients $a = c$ at large N_c. "No-go theorem" (?)
Bulk Weyl anomaly calculation always gives $a = c$ at leading order Henningson Skenderis

• Dual geometries are 10d

• Radius of curvature R related to coupling λ (a modulus),
$R \sim \lambda^{1/4}$, can be taken arbitrarily large (but $\lambda \longrightarrow 0$ not always an option)

't Hooft gave a very general heuristic argument for

"Large N field theory = closed string theory with $g_s \sim 1/N$"

So far we understand "well" only a limited class of dualities,
for the theories "in the universality class" of $\mathcal{N} = 4$ SYM

$\exists$ many string constructions of field theories with genuinely fewer d.o.f. in the IR (say pure $SU(N)$, or $\mathcal{N} = 1$ SYM).

However if one takes a limit that decouples the unwanted UV d.o.f,
the dual string is described (at best) by a strongly curved sigma model.

Hopefully this is just a technical problem, but progress has so far been limited.

Attack "next simplest case"

Ideal case study:

$\mathcal{N} = 2$ SYM with $N_f = 2N_c$ fundamental hypermultiplets

"$\mathcal{N} = 2$ SCQCD"

Large N limit à la Veneziano: $N_c \sim N_f$

• What (if any) is the dual string theory?

$\lambda = g_{YM}^2 N_c$ is an exactly marginal coupling, just as in $\mathcal{N} = 4$ SYM.

For large λ, a weakly curved gravity description?

String theory on... $AdS_5 \times \mathcal{X}$?

Long-standing open problem!

The Veneziano limit and dual strings

Focus on theories with large number of fundamental flavors, $N_f \sim N_c$.

Veneziano limit: $N_c \to \infty$, $N_f \to \infty$ with N_f/N_c fixed, $\lambda = g^2_{YM} N_c$ fix

Important applications to AdS/QCD.

Holography in the Veneziano limit?

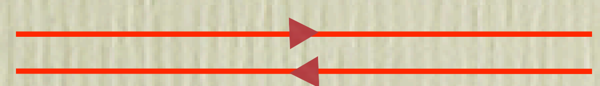
't Hooft argument for existence of dual closed string theory at large N
can be adapted to the Veneziano limit.

Schematically: adjoint fields $\phi^a{}_b$ $a = 1, \ldots, N_c$ color index

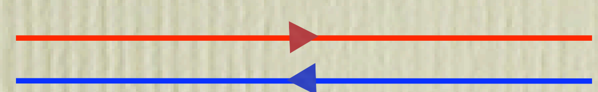
fundamental fields $q^a{}_i$ $i = 1, \ldots, N_f$ flavor index

Two kinds of double lines:

Adjoint lines $\langle \phi \, \phi \rangle$



Quark lines $\langle q \, q \rangle$



Quark lines *not* suppressed.

Vacuum Feynman diagrams $\longrightarrow$ bi-colored Riemann surfaces $\sim N^{2-2g}$
suggesting as usual a dual closed string theory with $g_s = 1/N$.

Main novelty: *glueball* operators $\mathrm{Tr}(\phi \dots \phi)$ (color-trace)

mix at leading order with

flavor-singlet *mesons* $\bar{q}^i \phi \dots \phi q_i$

Define flavor-contracted combination $\mathcal{M}^a{}_b \equiv q^a{}_i q^i{}_b$

In flavor-singlet sector, basic building blocks are the single-trace operators

$$\mathrm{Tr}(\phi^{k_1} \mathcal{M}^{l_1} \phi^{k_2} \mathcal{M}^{l_2} \dots)$$

Usual large N factorization arguments apply.

• In the (conjectural) dual string theory, large meson/glueball mixing interpreted as large backreaction of the "flavor" branes (need to resum open string perturbation theory).

Plan of attack

From the "bottom-up":

- Perturbative anomalous dimensions:

integrable spin-chain? asymptotic Bethe ansatz? emergent geometry?

- Spectrum of protected single-trace operators: KK spectrum?

- . . .

From the "top-down":

- Engineer it with branes in string theory

In both approaches, useful to consider more general family of superconformal theories, interpolating between a $\mathbb{Z}_2$ orbifold of $\mathcal{N} = 4$ and $\mathcal{N} = 2$ SCQCD

The Field Theory

$\mathcal{N}=2$ hypermultiplet

$$\begin{array}{ccc} & \psi_\alpha & \\ q & & \tilde{q}^* \\ & \bar{\tilde{\psi}}^{\dot\alpha} & \end{array}$$

$\mathcal{N}=2$ vector multiplet

$$\begin{array}{ccc} & A_\mu & \\ \lambda^1_\alpha & & \lambda^2_\alpha \\ & \phi & \end{array}$$

	$SU(N_c)$	$U(N_f)$	$SU(2)_R$	$U(1)_r$
$\mathcal{Q}^{\mathcal{I}}_\alpha$	**1**	**1**	**2**	$+1/2$
$\mathcal{S}_{\mathcal{I}\,\alpha}$	**1**	**1**	**2**	$-1/2$
A_μ	Adj	**1**	**1**	0
ϕ	Adj	**1**	**1**	-1
$\lambda^{\mathcal{I}}_\alpha$	Adj	**1**	**2**	$-1/2$
$Q_{\mathcal{I}}$	$\square$	$\square$	**2**	0
ψ_α	$\square$	$\square$	**1**	$+1/2$
$\tilde{\psi}_\alpha$	$\overline{\square}$	$\overline{\square}$	**1**	$+1/2$
$\mathcal{M}_{\mathbf{1}}$	Adj + **1**	**1**	**1**	0
$\mathcal{M}_{\mathbf{3}}$	Adj + **1**	**1**	**3**	0

$$S_V \;=\; -\int d^4x\,\mathrm{Tr}\Big(\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + i\,\bar{\lambda}_{\mathcal{I}}\bar{\sigma}^\mu D_\mu\lambda^{\mathcal{I}} + (D^\mu\phi)\,(D_\mu\phi)^\dagger + \sqrt{2}\,i\,g\,\epsilon_{\mathcal{I}\mathcal{J}}\lambda^{\mathcal{I}}\lambda^{\mathcal{J}}\phi^\dagger - \sqrt{2}\,i\,g\,\epsilon^{\mathcal{I}\mathcal{J}}\bar{\lambda}_{\mathcal{I}}\bar{\lambda}_{\mathcal{J}}\phi + \frac{g^2}{2}\,[\phi\,,\,\phi^\dagger]\Big)$$

$$S_H \;=\; -\int d^4x\Big(\big(D^\mu\bar{Q}^{\mathcal{I}}\big)\,(D_\mu Q_{\mathcal{I}}) + i\,\bar{\psi}\bar{\sigma}^\mu D_\mu\psi + i\,\tilde{\psi}\bar{\sigma}^\mu D_\mu\bar{\tilde{\psi}} + \sqrt{2}\,i\,g\,\epsilon^{\mathcal{I}\mathcal{J}}\bar{\psi}\bar{\lambda}_{\mathcal{I}}Q_{\mathcal{J}} - \sqrt{2}\,i\,g\,\epsilon_{\mathcal{I}\mathcal{J}}\bar{Q}^{\mathcal{I}}\lambda^{\mathcal{J}}\psi + \sqrt{2}\,i\,g\,\tilde{\psi}\lambda^{\mathcal{I}}Q_{\mathcal{I}} - \sqrt{2}\,i\,g\,\bar{Q}^{\mathcal{I}}\bar{\lambda}_{\mathcal{I}}\bar{\tilde{\psi}}$$

$$-2\,g^2\bar{Q}_{\mathcal{I}}\phi^\dagger\phi Q^{\mathcal{I}} + \sqrt{2}\,i\,g\,\tilde{\psi}\phi\psi - \sqrt{2}\,i\,g\,\bar{\psi}\bar{\phi}\bar{\tilde{\psi}} + g^2 V_Q\Big)$$

$SU(2)_R$ doublet $Q_{\mathcal{I}} = (q\,,\,\tilde{q}^*)$

Flavor-contracted mesonic operator: $\mathcal{M}^{\mathcal{I}}{}_{\mathcal{J}}{}^{a}{}_{b} = Q_{\mathcal{I}}{}^{a}{}_{i}\,\bar{Q}^{\mathcal{J}}_{b}{}^{i}$

$$\mathcal{M}_{\mathbf{1}} \equiv \mathcal{M}^{\mathcal{I}}{}_{\mathcal{I}} \quad \text{and} \quad \mathcal{M}_{\mathbf{3}} \equiv \mathcal{M}^{\mathcal{J}}_{\mathcal{K}} - \frac{1}{2}\mathcal{M}^{\mathcal{I}}{}_{\mathcal{I}}\delta^{\mathcal{J}}_{\mathcal{K}}$$

The squark potential is a function only of the triplet,

$$V_Q = \mathrm{Tr}\,(\mathcal{M}_{\mathbf{3}}\mathcal{M}_{\mathbf{3}}) - \frac{1}{N_c}\mathrm{Tr}\,(\mathcal{M}_{\mathbf{3}})\,\mathrm{Tr}\,(\mathcal{M}_{\mathbf{3}})$$

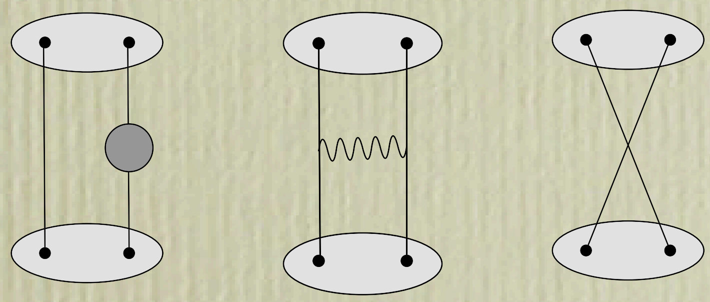
	$SU(N_c)$	$U(N_f)$	$SU(2)_R$	$U(1)_r$
$\mathcal{Q}^{\mathcal{I}}_{\alpha}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$+1/2$
$\mathcal{S}_{\mathcal{I}\,\alpha}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$-1/2$
A_μ	Adj	$\mathbf{1}$	$\mathbf{1}$	0
ϕ	Adj	$\mathbf{1}$	$\mathbf{1}$	-1
$\lambda^{\mathcal{I}}_{\alpha}$	Adj	$\mathbf{1}$	$\mathbf{2}$	$-1/2$
$Q_{\mathcal{I}}$	$\square$	$\square$	$\mathbf{2}$	0
ψ_α	$\square$	$\square$	$\mathbf{1}$	$+1/2$
$\tilde{\psi}_\alpha$	$\overline{\square}$	$\overline{\square}$	$\mathbf{1}$	$+1/2$
$\mathcal{M}_{\mathbf{1}}$	Adj $+\ \mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	0
$\mathcal{M}_{\mathbf{3}}$	Adj $+\ \mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	0

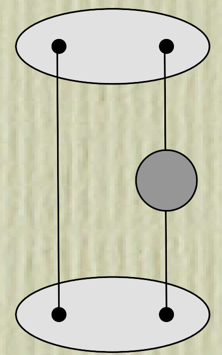
The One-Loop Hamiltonian in the Scalar Sector

We have evaluated the complete one-loop hamiltonian acting on single-trace operators made of scalars,

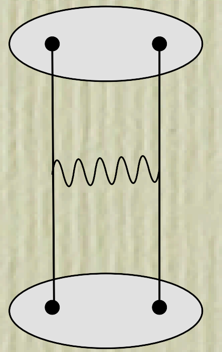
$$\mathrm{Tr}\left[\,\phi^k\bar{\phi}^\ell\mathcal{M}_{\mathbf{1}}^m\mathcal{M}_{\mathbf{3}}^n\,\right] \qquad \text{(arbitrary permutations thereof)}$$

Crucial observation: large N ensures locality of the hamiltonian.
Nearest neighbor at one-loop, next-to nearest at two loops, . . .
(Still true in the Veneziano limit).

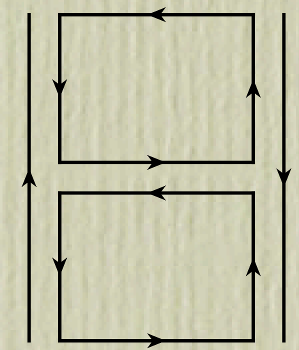




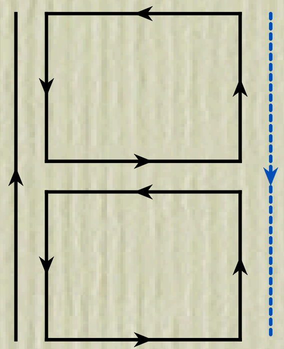
Wave function renormalization diagram



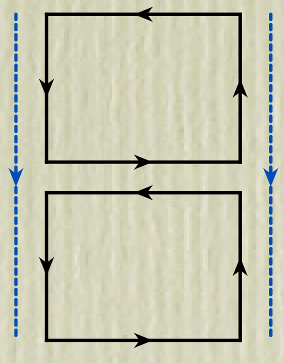
Gluon exchange diagrams



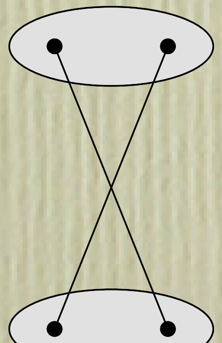
$\phi\phi \to \phi\phi$



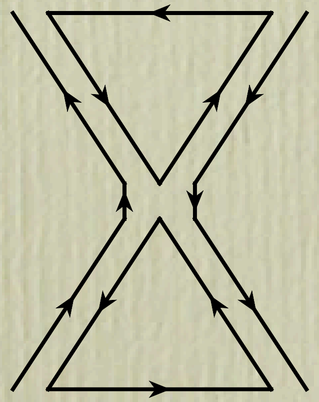
$\phi Q \to \phi Q$



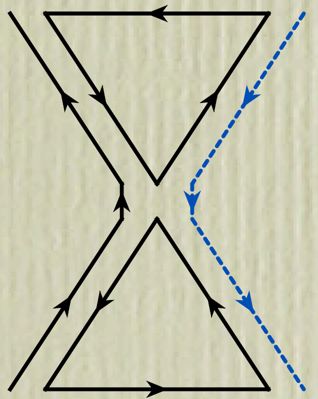
$QQ \to QQ$



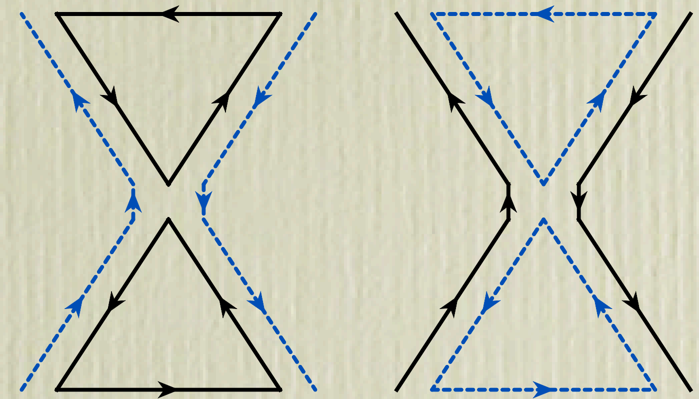
Quartic diagrams



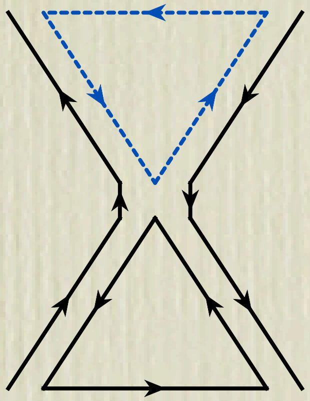
$\phi\phi \to \phi\phi$



$\phi Q \to \phi Q$



$QQ \to QQ$



$QQ \to \phi\phi$

Each site of the chain occupied by 6d vector space spanned by $\phi, \bar{\phi}, Q_{\mathcal{I}}, \bar{Q}^{\mathcal{J}}$.

Nearest neighbour Hamiltonian $H_{l,l+1}$ acting on $V_l \otimes V_{l+1}$

$\phi_{\mathfrak{m}} = (\phi, \bar{\phi})$

$$
\begin{array}{c|cccc}
 & \phi^{\mathfrak{p}}\phi^{\mathfrak{q}} & Q_{\mathcal{I}}\bar{Q}^{\mathcal{J}} & \bar{Q}^{\mathcal{K}}Q_{\mathcal{L}} & Q_{\mathcal{I}}\phi^{\mathfrak{p}} \\
\hline
\phi_{\mathfrak{p}'}\phi_{\mathfrak{q}'} & 2\delta^{\mathfrak{p}}_{\mathfrak{p}'}\delta^{\mathfrak{q}}_{\mathfrak{q}'} + g^{\mathfrak{p}\mathfrak{q}}g_{\mathfrak{p}'\mathfrak{q}'} - 2\delta^{\mathfrak{p}}_{\mathfrak{q}'}\delta^{\mathfrak{q}}_{\mathfrak{p}'} & \sqrt{\frac{N_f}{N_c}}g_{\mathfrak{p}'\mathfrak{q}'}\delta^{\mathcal{J}}_{\mathcal{I}} & 0 & 0 \\
\bar{Q}^{\mathcal{I}'}Q_{\mathcal{J}'} & \sqrt{\frac{N_f}{N_c}}g^{\mathfrak{p}\mathfrak{q}}\delta^{\mathcal{I}'}_{\mathcal{J}'} & (2\delta^{\mathcal{I}'}_{\mathcal{I}}\delta^{\mathcal{J}}_{\mathcal{J}'} - \delta^{\mathcal{J}}_{\mathcal{I}}\delta^{\mathcal{I}'}_{\mathcal{J}'})\frac{N_f}{N_c} & 0 & 0 \\
Q_{\mathcal{K}'}\bar{Q}^{\mathcal{L}'} & 0 & 0 & 2\delta^{\mathcal{K}}_{\mathcal{L}}\delta^{\mathcal{L}'}_{\mathcal{K}'} & 0 \\
\bar{Q}^{\mathcal{I}'}\phi_{\mathfrak{p}'} & 0 & 0 & 0 & 2\delta^{\mathcal{I}'}_{\mathcal{I}}\delta^{\mathfrak{p}}_{\mathfrak{p}'}
\end{array}
$$

$SU(2)_R$ indices $\mathcal{I}, \mathcal{J}, \mathcal{K}, \mathcal{L} \dots = 1, 2$ $\qquad$ $U(1)_r$ indices $\mathfrak{m}, \mathfrak{n} \dots = 1, 2$ $\qquad$ $g_{\mathfrak{m}\mathfrak{n}} = g^{\mathfrak{m}\mathfrak{n}} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$$\Gamma^{(1)} \equiv g^2 H \,, \qquad g^2 \equiv \frac{\lambda}{8\pi^2} \,, \quad \lambda \equiv g_{YM}^2 N_c$$

Elementary operators acting on each site of the chain, transforming "incoming" $\mathcal{O}^{\mathcal{I}}{}_{\mathcal{J}}$ to "outgoing" $\mathcal{O}^{\mathcal{L}}{}_{\mathcal{K}}$:

Trace operator $\mathbb{K}_{\mathcal{I}\mathcal{K}}^{\mathcal{J}\mathcal{L}} = \delta_{\mathcal{I}}^{\mathcal{J}} \delta_{\mathcal{K}}^{\mathcal{L}}$

Permutation operator $\mathbb{P}_{\mathcal{I}\mathcal{K}}^{\mathcal{J}\mathcal{L}} = \delta_{\mathcal{I}\mathcal{K}} \delta^{\mathcal{J}\mathcal{L}}$

Identity operator $\mathbb{I}_{\mathcal{I}\mathcal{K}}^{\mathcal{J}\mathcal{L}} = \delta_{\mathcal{I}}^{\mathcal{L}} \delta_{\mathcal{K}}^{\mathcal{J}}$

$$H_{k,k+1} = \begin{array}{c} \\ \phi\phi \\ \bar{Q}Q \\ Q\bar{Q} \\ \bar{Q}\phi \end{array} \begin{array}{c} \begin{array}{cccc} \phi\phi & Q\bar{Q} & \bar{Q}Q & Q\phi \end{array} \\ \left(\begin{array}{cccc} 2\mathbb{I} + \mathbb{K} - 2\mathbb{P} & \sqrt{\frac{N_f}{N}}\,\mathbb{K} & 0 & 0 \\ \sqrt{\frac{N_f}{N}}\,\mathbb{K} & (2\mathbb{I} - \mathbb{K})\,\frac{N_f}{N_c} & 0 & 0 \\ 0 & 0 & 2\mathbb{K} & 0 \\ 0 & 0 & 0 & 2\mathbb{I} \end{array} \right) \end{array}$$

This spin-chain hamiltonian appears to be new.

Vacuum $\mathrm{Tr}(\phi^\ell)$.

Study excitations above the vacuum in the language of the asymptotic Bethe ansatz.

In the one-impurity sector:

$$\bar\phi(p) \equiv \sum_x \bar\phi(x) e^{ipx}\,, \quad \mathcal{M}_{\mathbf{1}}(p) \equiv \sum_x \mathcal{M}_{\mathbf{1}}(x) e^{ipx}$$

$$H \begin{pmatrix} \bar\phi(p) \\ \mathcal{M}_{\mathbf{1}} \end{pmatrix} = \begin{pmatrix} 6 - e^{ip} - e^{-ip} & (1+e^{-ip})\sqrt{\frac{2N_f}{N_c}} \\ (1+e^{ip})\sqrt{\frac{2N_f}{N_c}} & 4 \end{pmatrix} \begin{pmatrix} \bar\phi(p) \\ \mathcal{M}_{\mathbf{1}} \end{pmatrix}$$

For $N_f = 2N_c$ one of the two excitations is gapless

The chain is gapless for $N_f = 2N_c$!

The $N_f = 0$ case has been considered before. Di Vecchia–Tanzini

Light magnons correspond to the propagation of $T \equiv \phi\bar\phi - \mathcal{M}_{\mathbf{1}}$ along the chain.

For $N_f = 2N_c$, zero-momentum state $\mathrm{Tr}T\phi^\ell$ has zero anomalous dimension.

Protected Operators

From explicit one-loop calculation in the scalar sector, the single-trace operators with $\gamma = 0$ are

- $\mathrm{Tr}\,\mathcal{M}_{\mathbf{3}}$
- $\mathrm{Tr}\,\phi^\ell$, with $\ell \geq 2$.
- $\mathrm{Tr}\,T\,\phi^\ell$, with $\ell \geq 0$, where $T \equiv \bar{\phi}\phi - \mathcal{M}_{\mathbf{1}}$.

Scalar Multiplets	SCQCD operators	Protected
$\mathcal{B}_{R,r(0,0)}$	$\mathrm{Tr}[\bar{\phi}^r \mathcal{M}_3^R]$	
$\mathcal{E}_{r(0,0)}$	$\mathrm{Tr}[\bar{\phi}^r]$	✓
$\hat{\mathcal{B}}_R$	$\mathrm{Tr}[\mathcal{M}_3^R]$	✓ for $R = 1$
$\mathcal{C}_{R,r(0,0)}$	$\mathrm{Tr}[T\mathcal{M}_3^R\bar{\phi}^r]$	
$\mathcal{C}_{0,r(0,0)}$	$\mathrm{Tr}[T\bar{\phi}^r]$	✓
$\hat{\mathcal{C}}_{R(0,0)}$	$\mathrm{Tr}[T\mathcal{M}_3^R]$	
$\hat{\mathcal{C}}_{0(0,0)}$	$\mathrm{Tr}[T]$	✓
$\mathcal{D}_{R(0,0)}$	$\mathrm{Tr}[\mathcal{M}_3^R\bar{\phi}]$	

Note that $\mathrm{Tr}\,T$ $(\Delta = 2)$ is the lowest weight state of the $\mathcal{N} = 2$ stress-tensor multiplet.

These operators are superconformal primaries.
In the free theory they are the lowest weight states of (semi-)short multiplets.
In the interacting theory (semi-)short multiplets can a priori combine into long multiplets with $\gamma \neq 0$.

Protection of $\mathrm{Tr}\phi^\ell$ easily proved to all orders from superconformal representation theory: such multiplets never appear in decomposition of long multiplets. Dolan-Osborn

Protection of $\mathrm{Tr}\, \mathcal{M}_{\mathbf{3}}$ and of $\mathrm{Tr}\, T\, \phi^{\ell}$ more subtle,
we prove it by computing (essentially) a superconformal index.
Most easily done in interpolating family of SCFTs (coming up soon).

(Situations more intricate than in $\mathcal{N} = 4$ SYM where the only single-trace protected multiplets are the 1/2 BPS multiplets.)

There are no other single-trace protected multiplets.

6d geometry??

- $\text{Tr}\,\mathcal{M}_\mathbf{3}$
- $\text{Tr}\,\phi^\ell$, with $\ell \geq 2$.
- $\text{Tr}\,T\,\phi^\ell$, with $\ell \geq 0$, where $T \equiv \bar{\phi}\phi - \mathcal{M}_\mathbf{1}$.

Protected operators strongly suggestive of a supergravity spectrum from Kaluza-Klein on S^1

Remarkably, $\{\,\text{Tr}\,\mathcal{M}_\mathbf{3}\,, \text{Tr}\,\phi^\ell\,\}$ can be exactly matched to

KK reduction of 6d $(4,0)$ tensor multiplet on $AdS_5 \times S^1$!

However there is no simple 6d origins for the $\{\,\text{Tr}\,T\,\phi^\ell\,\}$ states.

KK reduction on S^1 of 6d $(4,0)$ supergravity multiplet can yield only a subset of $\{\,\text{Tr}\,T\,\phi^\ell\,\}$.

(At any rate a 6d $(4,0)$ sugra theory would be problematic for anomaly cancellation).

KK reduction of (4,0) tensor multiplet on $AdS_5 \times S^1$

Field Theory				Gravity		
Operator	k	$U(1)_r$	Δ	Mass	Field	KK
$\mathrm{Tr}[\lambda\lambda\bar{\phi}^{k-1}]$	$k \geq 1$	k	$2+k$	k^2-4	ξ^i	k
$\mathrm{Tr}[F^2\bar{\phi}^k]$	$k \geq 0$	k	$4+k$	k^2+4k	ξ	$k+1$
$\mathrm{Tr}[\bar{\phi}^k]$	$k \geq 2$	k	k	k^2-4k	$\bar{\xi}$	$k-1$
$\mathrm{Tr}[F\bar{\phi}^k]$	$k \geq 1$	k	$2+k$	k^2	$B^-_{\hat{m}\hat{n}}$	k

(From Gukov with minor modification for zero modes).

Table shows correspondence of positive ($k \geq 1$) KK modes of tensor multiplet with field theory operators: exact matching with $\mathrm{Tr}\,\bar{\phi}^\ell$ multiplets, with $\ell = k+1$.

Zero-modes on S^1 match with $\mathrm{Tr}\,\mathcal{M}_3$ multiplet.

$\mathrm{Tr}\,\phi^\ell$ multiplet

Δ					
ℓ	$0_{(0,0)}$				
$\ell+1/2$		$\frac{1}{2}{}_{(0,\pm\frac{1}{2})}$			
$\ell+1$			$0_{(0,\pm1)}, 1_{(0,0)}$		
$\ell+3/2$				$\frac{1}{2}{}_{(0,\pm\frac{1}{2})}$	
$\ell+2$					$0_{(0,0)}$
r	$-\ell$	$-\ell+1/2$	$-\ell+1$	$-\ell+3/2$	$-\ell+2$

$\mathcal{M}_3$ multiplet

Δ					
2			$1_{(0,0)}$		
$5/2$		$\frac{1}{2}{}_{(\frac{1}{2},0)}$		$\frac{1}{2}{}_{(0,\frac{1}{2})}$	
3	$0_{(0,0)}$		$0_{(\frac{1}{2},\frac{1}{2})}$		$0_{(0,0)}$
$7/2$					
4			$-0_{(0,0)}$		
r	-1	$-1/2$	0	$1/2$	1

$\mathbf{Tr}\, T\phi^{\ell}$ multiplet

Δ							
$2+\ell$			$0_{(0,0)}$				
$5/2+\ell$		$\frac{1}{2}_{(\frac{1}{2},0)}$		$\frac{1}{2}_{(0,\frac{1}{2})}$			
$3+\ell$	$0_{(1,0)}$		$1_{(\frac{1}{2},\frac{1}{2})}$, $0_{(\frac{1}{2},\frac{1}{2})}$		$0_{(0,1)}$, $1_{(0,0)}$		
$7/2+\ell$		$\frac{1}{2}_{(1,\frac{1}{2})}$		$\frac{1}{2}_{(\frac{1}{2},0)}$, $\frac{3}{2}_{(\frac{1}{2},0)}$, $\frac{1}{2}_{(\frac{1}{2},1)}$		$\frac{1}{2}_{(0,\frac{1}{2})}$	
$4+\ell$			$1_{(1,0)}$, $0_{(1,1)}$		$0_{(\frac{1}{2},\frac{1}{2})}$, $1_{(\frac{1}{2},\frac{1}{2})}$		$0_{(0,0)}$
$9/2+\ell$				$\frac{1}{2}_{(1,\frac{1}{2})}$		$\frac{1}{2}_{(\frac{1}{2},0)}$	
$5+\ell$					$0_{(1,0)}$		
r	$-\ell-1$	$-\ell-1/2$	$-\ell$	$-\ell+1/2$	$-\ell+1$	$-\ell+3/2$	$-\ell+2$

An interpolating family of super CFTs

$\mathcal{N}=2$ SCQCD can be viewed as a limit of a family of $\mathcal{N}=2$ SCFTs.

In opposite limit the family reduces to a well-known $\mathbb{Z}_2$ orbifold of $\mathcal{N}=4$ SYM

Start with $\mathcal{N}=4$ SYM: X_{AB}, λ_α^A, A_μ

A, B $SU(4)_R$ indices

$$X_{AB}=\frac{1}{\sqrt{2}}\left(\begin{array}{cc|cc} 0 & X_4+iX_5 & X_7+iX_6 & X_8+iX_9 \\ -X_4-iX_5 & 0 & X_8-iX_9 & -X_7+iX_6 \\ \hline -X_7-iX_6 & -X_8+iX_9 & 0 & X_4-iX_5 \\ -X_8-iX_9 & X_7-iX_6 & -X_4+iX_5 & 0 \end{array}\right)$$

Pick $SU(2)_L\times SU(2)_R\times U(1)_r$ subgroup of $SU(4)_R$

$$\begin{array}{c} 1\ + \\ 2\ - \\ 3\ \hat{+} \\ 4\ \hat{-} \end{array}\left(\begin{array}{c|c} SU(2)_R\times U(1)_r & \\ & \\ \hline & \\ & SU(2)_L\times U(1)_r^* \end{array}\right)$$

$$\mathcal{Z}\equiv\frac{X_4+iX_5}{\sqrt{2}}\,,\qquad \mathcal{X}_{\mathcal{I}\hat{\mathcal{I}}}\equiv\frac{1}{\sqrt{2}}\begin{pmatrix} X_7+iX_6 & X_8+iX_9 \\ X_8-iX_9 & -X_7+iX_6 \end{pmatrix}$$

$\mathcal{I},\mathcal{J}=\pm$ $SU(2)_R$ indices, $\hat{\mathcal{I}},\hat{\mathcal{J}}=\hat{\pm}$ $SU(2)_L$ indices

$SU(2)_L\times SU(2)_R\cong SO(4)$ are 6789 rotations,

$U(1)_R\cong SO(2)$ 45 rotations.

In R-space, orbifold by $\mathbb{Z}_2 \subset SU(2)_L$, $\mathbb{Z}_2 = \{\pm \mathbb{I}_{2\times 2}\}$

$$(X_6, X_7, X_8, X_9) \to \pm (X_6, X_7, X_8, X_9)$$

In color space, start with $SU(2N_c)$ and declare non-trivial element of orbifold

$$\tau \equiv \begin{pmatrix} \mathbb{I}_{N_c\times N_c} & 0 \\ 0 & -\mathbb{I}_{N_c\times N_c} \end{pmatrix}$$

$$A_\mu \to \tau A_\mu \tau\,, \quad Z_{\mathcal{I}\mathcal{J}} \to \tau Z_{\mathcal{I}\mathcal{J}} \tau\,, \quad \lambda_{\mathcal{I}} \to \tau \lambda_{\mathcal{I}} \tau\,, \quad \mathcal{X}_{\mathcal{I}\hat{\mathcal{I}}} \to -\tau \mathcal{X}_{\mathcal{I}\hat{\mathcal{I}}} \tau\,, \quad \lambda_{\hat{\mathcal{I}}} \to -\tau \lambda_{\hat{\mathcal{I}}} \tau$$

$$A_\mu = \begin{pmatrix} A^a_{\mu b} & 0 \\ 0 & \check{A}^{\check{a}}_{\mu \check{b}} \end{pmatrix} \qquad Z = \begin{pmatrix} \phi^a{}_b & 0 \\ 0 & \check{\phi}^{\check{a}}{}_{\check{b}} \end{pmatrix} \qquad \lambda_{\mathcal{I}} = \begin{pmatrix} \lambda^a_{\mathcal{I}b} & 0 \\ 0 & \check{\lambda}^{\check{a}}_{\mathcal{I}\check{b}} \end{pmatrix}$$

$$\lambda_{\hat{\mathcal{I}}} = \begin{pmatrix} 0 & \psi^a_{\hat{\mathcal{I}}\check{a}} \\ \tilde{\psi}^{\check{b}}_{\hat{\mathcal{I}}b} & 0 \end{pmatrix} \qquad \mathcal{X}_{\mathcal{I}\hat{\mathcal{I}}} = \begin{pmatrix} 0 & Q_{\mathcal{I}\hat{\mathcal{I}}\check{a}}{}^{a} \\ -\epsilon_{\mathcal{I}\mathcal{J}}\epsilon_{\hat{\mathcal{I}}\hat{\mathcal{J}}}\bar{Q}^{\check{b}\hat{\mathcal{J}}\mathcal{J}}{}_{b} & 0 \end{pmatrix}$$

	$SU(N_c)_1$	$SU(N_c)_2$	$SU(2)_R$	$SU(2)_L$	$U(1)_R$
$\mathcal{Q}^{\mathcal{I}}_{\alpha}$	**1**	**1**	**2**	**1**	+1/2
$\mathcal{S}_{\mathcal{I}\,\alpha}$	**1**	**1**	**2**	**1**	–1/2
A_μ	Adj	**1**	**1**	**1**	0
$\check{A}_\mu$	**1**	Adj	**1**	**1**	0
ϕ	Adj	**1**	**1**	**1**	–1
$\check{\phi}$	**1**	Adj	**1**	**1**	–1
$\lambda^{\mathcal{I}}$	Adj	**1**	**2**	**1**	–1/2
$\check{\lambda}^{\mathcal{I}}$	**1**	Adj	**2**	**1**	–1/2
$Q_{\mathcal{I}\hat{\mathcal{I}}}$	$\square$	$\overline{\square}$	**2**	**2**	0
$\psi_{\hat{\mathcal{I}}}$	$\square$	$\overline{\square}$	**1**	**2**	+1/2
$\tilde{\psi}_{\hat{\mathcal{I}}}$	$\overline{\square}$	$\square$	**1**	**2**	+1/2

$$
\begin{aligned}
\mathcal{L}_{Yukawa}(g_{YM},\check{g}_{YM}) = i\sqrt{2}\mathrm{Tr}\big[& -g_{YM}\epsilon^{\mathcal{I}\mathcal{J}}\bar{\lambda}_{\mathcal{I}}\bar{\lambda}_{\mathcal{J}}\phi - \check{g}_{YM}\epsilon^{\mathcal{I}\mathcal{J}}\bar{\check{\lambda}}_{\mathcal{I}}\bar{\check{\lambda}}_{\mathcal{J}}\check{\phi} \\
& +g_{YM}\epsilon^{\hat{\mathcal{I}}\hat{\mathcal{J}}}\tilde{\psi}_{\hat{\mathcal{I}}}\phi\psi_{\hat{\mathcal{J}}} + \check{g}_{YM}\epsilon^{\hat{\mathcal{I}}\hat{\mathcal{J}}}\psi_{\hat{\mathcal{J}}}\check{\phi}\tilde{\psi}_{\hat{\mathcal{I}}} \\
& +g_{YM}\epsilon^{\hat{\mathcal{I}}\hat{\mathcal{J}}}\tilde{\psi}_{\hat{\mathcal{J}}}\lambda^{\mathcal{I}}Q_{\mathcal{I}\hat{\mathcal{I}}} + \check{g}_{YM}\epsilon^{\hat{\mathcal{I}}\hat{\mathcal{J}}}Q_{\mathcal{I}\hat{\mathcal{I}}}\check{\lambda}^{\mathcal{I}}\tilde{\psi}_{\hat{\mathcal{J}}} \\
& -g_{YM}\epsilon_{\mathcal{I}\mathcal{J}}\bar{Q}^{\hat{\mathcal{J}}\mathcal{I}}\lambda^{\mathcal{J}}\psi_{\hat{\mathcal{J}}} - \check{g}_{YM}\epsilon_{\mathcal{I}\mathcal{J}}\psi_{\hat{\mathcal{J}}}\check{\lambda}^{\mathcal{I}}\bar{Q}^{\hat{\mathcal{J}}\mathcal{J}}\big] + h.c.
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{V}(g_{YM},\check{g}_{YM}) = \; & g^2_{YM}\mathrm{Tr}\big[\frac{1}{2}[\bar{\phi},\phi]^2 + \mathcal{M}_{\mathcal{I}}^{\;\mathcal{I}}(\phi\bar{\phi}+\bar{\phi}\phi) + \mathcal{M}_{\mathcal{I}}^{\;\mathcal{J}}\mathcal{M}_{\mathcal{J}}^{\;\mathcal{I}} - \frac{1}{2}\mathcal{M}_{\mathcal{I}}^{\;\mathcal{I}}\mathcal{M}_{\mathcal{J}}^{\;\mathcal{J}} \\
& +\check{g}^2_{YM}\mathrm{Tr}\big[\frac{1}{2}[\bar{\check{\phi}},\check{\phi}]^2 + \check{\mathcal{M}}^{\mathcal{I}}{}_{\mathcal{I}}(\check{\phi}\bar{\check{\phi}}+\bar{\check{\phi}}\check{\phi}) + \check{\mathcal{M}}^{\mathcal{I}}{}_{\mathcal{J}}\check{\mathcal{M}}^{\mathcal{J}}{}_{\mathcal{I}} - \frac{1}{2}\check{\mathcal{M}}^{\mathcal{I}}{}_{\mathcal{I}}\check{\mathcal{M}}^{\mathcal{J}} \\
& +g_{YM}\check{g}_{YM}\mathrm{Tr}\big[-2Q_{\mathcal{I}\hat{\mathcal{I}}}\check{\phi}\bar{Q}^{\hat{\mathcal{I}}\mathcal{I}}\bar{\phi} + h.c.\big] - \frac{1}{N_c}\mathcal{V}_{d.t.}\,,
\end{aligned}
$$

Two gauge-couplings g_{YM} and $\check{g}_{YM}$ can be independently varied

while preserving $\mathcal{N}=2$ superconformal invariance

For $\check{g}_{YM} \to 0$, recover $\mathcal{N}=2$ SCQCD $\oplus$ decoupled $SU(N_{\check{c}})$ vector multiplet

For $\check{g}_{YM}=0$, global symmetry enhancement $SU(N_{\check{c}}) \times SU(2)_L \to U(N_f = 2N_c)$:
$(\check{a}, \hat{\mathcal{I}}) \equiv i = 1, \ldots N_f = 2N_c$

$$c = a = \frac{N_c^2}{2}$$ along the whole marginal deformation

For $\check{g}_{YM}=0$, interpret as

$$a = \left(\frac{7}{24} + \frac{5}{24}\right) N_c^2 \qquad c = \left(\frac{8}{24} + \frac{4}{24}\right) N_c^2$$

Spin-chain for interpolating family

$$H_{k,k+1} =$$

$$\begin{array}{c} \\ \phi\phi \\ Q\bar{Q} \\ \check{\phi}\check{\phi} \\ \bar{Q}Q \\ \phi Q \\ Q\check{\phi} \\ \check{\phi}\bar{Q} \\ \bar{Q}\phi \end{array}
\begin{array}{c} \begin{array}{cccccccc} \phi\phi & Q\bar{Q} & \check{\phi}\check{\phi} & \bar{Q}Q & \phi Q & Q\check{\phi} & \check{\phi}\bar{Q} & \bar{Q}\phi \end{array} \\
\begin{pmatrix}
g^2(2+\mathbb{K}-2\mathbb{P}) & g^2\mathbb{K} & 0 & 0 & 0 & 0 & 0 & 0 \\
g^2\mathbb{K} & g^2(2-\mathbb{K})\hat{\mathbb{K}}+2\check{g}^2\mathbb{K} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \check{g}^2(2+\mathbb{K}-2\mathbb{P}) & \check{g}^2\mathbb{K} & 0 & 0 & 0 & 0 \\
0 & 0 & \check{g}^2\mathbb{K} & \check{g}^2(2-\mathbb{K})\hat{\mathbb{K}}+2g^2\mathbb{K} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 2g^2 & -2g\check{g} & 0 & 0 \\
0 & 0 & 0 & 0 & -2g\check{g} & 2\check{g}^2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 2\check{g}^2 & -2g\check{g} \\
0 & 0 & 0 & 0 & 0 & 0 & -2g\check{g} & 2g^2
\end{pmatrix} \end{array}$$

Spin-chain has very interesting dynamics.

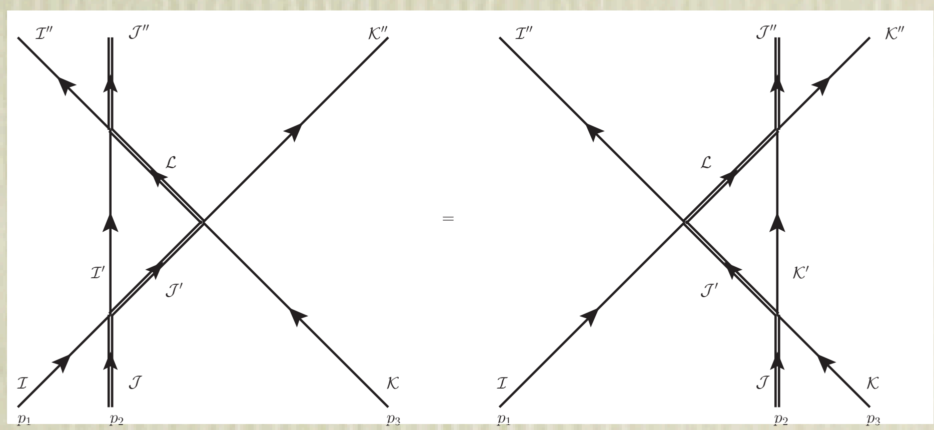
S-matrix factorizes into "left" and "right"

$$S_{Q\bar{Q}}(p_1, p_2, \kappa) = -S_L(p_1, p_2, \kappa) S_R(p_1, p_2, \kappa) \qquad \kappa \equiv \check{g}/g.$$

$SU(2)_L$	$S_L(p_1, p_2, \kappa)$	$SU(2)_R$	$S_R(p_1, p_2, \kappa)$
1_L	$\mathcal{S}(p_1, p_2, \kappa - \frac{1}{\kappa})$	1_R	$\mathcal{S}^{-1}(p_1, p_2, \kappa)$
3_L	$\mathcal{S}(p_1, p_2, \kappa)$	3_R	-1

$$\mathcal{S}(p_1, p_2, \kappa) \equiv -\frac{1 - 2\kappa e^{ip_1} + e^{i(p_1+p_2)}}{1 - 2\kappa e^{ip_2} + e^{i(p_1+p_2)}}$$

Yang-Baxter fails for $\check{g} \neq g$, but holds again for $\check{g} = 0$!



"Dimeric" excitations $T(p)$, $\widetilde{T}(p)$ and $\mathcal{M}_3$ emerge smoothly as bound states as $\check{g} \longrightarrow 0$

Interpolating theory has vastly more protected "closed" states than $\mathcal{N} = 2$ SCQD:

towers of states with arbitrary high (and equal) $SU(2)_L$ and $SU(2)_R$ spins

For $\check{g} \to 0$, they are re-interpreted as multiparticle states of short open strings

Top-down: embedding in string theory

Well–known realizations of interpolating SCFT in string theory.

$\mathbb{Z}_2$ orbifold of $\mathcal{N} = 4$ SYM realized on N_c D3 branes on $\mathbb{R}^4/\mathbb{Z}_2 \times \mathbb{R}^2$ Douglas Moore

Near-horizon geometry is the familiar background $AdS_5 \times S^5/\mathbb{Z}_2$ Kachru-Silverstein

Varying relative couplings corresponds to changing period of B_{NS} on collapsed S^2:

at orbifold point $g = \check{g}$ and $\int_{S^2} B_{NS} = 1/2$.

As $\check{g} \to 0$, $\int B_{NS} \to 0$: singular Calabi-Yau.

More useful to T-dualize to Hanany-Witten setup in Type IIA:

Two stacks of N_c D4s suspended between two NS5 branes

NS5	012345
D4	0123 6

with $x_6 \sim x_6 + 2\pi R_6$.

Interpolating family of theories parametrized by $\gamma = L_6/R_6$, where L_6 is the distance between the two NS5s.

- For $\gamma = \pi$, $\mathbb{Z}_2$ orbifold of $\mathcal{N} = 4$ SYM:

Varying γ, exactly marginal deformation of the $\mathbb{Z}_2$ orbifold.

Two gauge couplings

$$\frac{1}{g^2} = \frac{\gamma R_6}{g_s l_s} \qquad \frac{1}{\check{g}^2} = \frac{(2\pi - \gamma) R_6}{g_s l_s}$$

$$\frac{1}{g^2} + \frac{1}{\check{g}^2} = \frac{2\pi R_6}{g_s l_s} \qquad \frac{\check{g}^2}{g^2} = \frac{\gamma}{2\pi - \gamma} \,.$$

Decoupling limit

$$g_s \to 0 \,, \quad l_s \to 0 \,, \quad R_6 \to 0$$

with g, $\check{g}$ fixed.

Hierarchy of scales

$$L \gg l_s \gg R_6 \geq L_6 \equiv \gamma R_6$$

where L is the length above which field theory description is valid.

• T duality around x_6: N_c D3 branes on

2-center Taub-Nut $\times \mathbb{R}^2 \xrightarrow{R_6 \to 0} \mathbb{R}^4/\mathbb{Z}_2 \times \mathbb{R}^2$.

The angle γ is mapped to the B_{NS} flux the on collapsed S^2,

$$\int_{S^2} B_{NS} = \frac{\gamma}{2\pi}$$

Indeed $\int B_{NS} = 1/2$ at the CFT orbifold point Aspinwall

Note also that

$$g_s^{IIB} = \frac{l_s}{R_6} g_s^{IIA} = g^2 \gamma$$

As $\gamma \to 0$, we can focus on local singularity.

Precisely in the limit we are interested in, correct description of the two NS5 branes is in terms of

8d non-critical string theory

with exact CFT (after an angular T-duality) $\mathbb{R}^{(5,1)} \times SL(2)_2/U(1)$ Giveon-Kutasov

Recall that for k NS5 branes on a circle double-scaling limit gives $(SL(2)_k/U(1) \times SU(2)_k/U(1))/\mathbb{Z}_k$
The piece of the geometry which is "lost" in $10d \to 8d$ is the $SU(2)/U(1)$ factor of the CFT,
which has $c = 0$ for $k = 2$ NS5s.

Holographically the loss of a piece of the geometry (and of associated KK tower) gets related to the decoupling of the extra vector multiplet and restriction to $SU(2)_L$ singlets.

Symmetry enhancement in cigar CFT for $k = 2$:
circle at free-fermion radius, $SU(2) \times SU(2)$ current algebra, broken to the diagonal $SU(2)$ by cigar interaction:
interpreted as 789 rotations in HW setup

Nice understanding of D-branes on the $SL(2)/U(1)$ cigar CFT

Israel-Pakman-Troost, Ashok-Murthy-Troost, Fotopulos-Niarchos-Prezas, ...

Compact ("color") D4s $\longrightarrow$ branes localized at the tip of the cigar, filling $\mathbb{R}^{(3,1)} \subset \mathbb{R}^{(5,1)}$

Non-compact ("flavor") D4s $\longrightarrow$ branes filling the cigar and $\mathbb{R}^{(3,1)} \subset \mathbb{R}^{(5,1)}$

Precisely for $N_f = 2N_c$ the dilaton tadpole of combined brane system cancels Murthy Troost

We are interested in the background after the backreaction of the branes.

Exact RR σ model still not known. Plausible ansatz:

$$ds^2 = f(\alpha)\left[\frac{dr^2}{r^2} + r^2 dx_\mu dx^\mu\right] + d\alpha^2 + g(\alpha)d\theta^2 + h(\alpha)d\varphi^2$$

with θ 45 angle, φ cigar angle, $r^2 = r_{45}^2 + r_{cigar}^2$, $\tan\alpha = r_{45}/r_{cigar}$.

Expect $f \sim g \gg h \sim l_s^2$ for generic α, and $g(\alpha = 0) = 0$.

(Note that before backreaction D5 ("flavor") branes wrap φ and are localized at $\alpha = 0$).

General features seem to match nicely bottom-up expectations:

• R-symmetry:
$U(1)_r$ symmetry from 45 isometry (large circle which gives KK modes),
$SU(2)_R$ comes stringy cigar symmetry ($SU(2)_R$ gauge-fields only)

• Spacetime susy: cigar $\times \mathbb{R}^{(5,1)}$ has $(4,0)$ (16 supercharges) on Minkowski directions
Branes break 1/2, near-horizon doubles again to 16 Supercharges = 8 Q + 8 S

• Closed string spectrum on non-critical IIB on $\mathbb{R}^{(5,1)} \times SL(2)_2/U(1)$:
(i) Normalizable modes localized at tip of the cigar:
6d tensor $(4,0)$ multiplet $\leftrightarrow \{\,\mathrm{Tr}\phi^l,\, \mathrm{Tr}\boldsymbol{\mathcal{M}_3}\,\}$

(ii) Delta-function normalizable modes propagating in the bulk.
Naively "massive", but lowest modes on S^1 of cigar (*e.g.* graviton) have in fact "massless" 7d gauge invariance

Richer spectrum that just 6d graviton multiplet. Its KK reduction on large S^1 should match with $\{\,\mathrm{Tr}T\phi^\ell\,\}$, but need back-reacted geometry

Explanation of the anomaly puzzle?

• For the whole interpolating family, $c = a$ after all!

As $\check{g} \to 0$, second vector multiplet becomes free but still contributes to the anomaly. In the dual bulk theory it must correspond to $\sim N_c^2$ decoupled "singleton" d.o.f.

So if we keep the spectator vector multiplet, it is consistent to assume that the dual theory has standard Einstein-Hilbert term in AdS_5.

• Alternatively, we may wish to discuss the theory [illegible] decoupled vector multiplet.

Speculation: integrating out the singletons generate the Chern-Simons term (and its susy completion) that contributed to $c - a$.

Conclusions

Natural speculation:

4d QFTs with lower (genuinely) lower susy are holographic to non-critical strings

$\mathcal{N} = 4$: 10d, critical case Maldacena

$\mathcal{N} = 2$: 8d (this talk)

$\mathcal{N} = 1$: 6d Klebanov Maldacena

$\mathcal{N} = 0$: 5d? Polyakov

We gave evidence for $\mathcal{N} = 2$ case, in simplest theory beyond "$\mathcal{N} = 4$ universality class":

Bottom-up (one-loop hamiltonian, superconformal representation theory)
and the top-down (string theory) suggest duality

$\mathcal{N} = 2$ SYM SCQCD $\leftrightarrow$ non-critical 8d IIB string theory with large $AdS_5 \times S^1$

Still a lot of work to do!

• Integrability of our spin chain (for $\check{g} = 0$)? asymptotic Bethe ansatz?

• Magnon S-matrix from dual sigma model
Certainly doable around orbifold point. Integrability not essential for comparison.

• Transition between critical and non-critical string

• Precise σ model with RR flux:
from supercoset construction?

• Implications for $\mathcal{N} = 1$ SQCD?

• ......

Many new possibilities.