THE CHOICE OF BASIS FOR PROJECTION METHODS IN WEAKLY SINGULAR INTEGRAL EQUATIONS

FILOMENA D. D' ALMEIDA

Centro de Matemática da Universidade do Porto (CMUP)and Faculdade de Engenharia da Universidade do Porto

Rua Dr. Roberto Frias s/n, 4200-265 Porto, Portugal

E-mail: `falmeida@fe.up.pt`

When considering a weakly singular Fredholm integral equation of the 2nd kind

$$T\varphi - z\varphi = f, \tag{1}$$

where z is in the resolvent set of T, $f \in X$, and $T : X \to X$, is a compact linear integral operator on the space of Lebesgue integrable complex valued functions X, defined by

$$(T\varphi)(\tau) = \int_0^{\tau^*} g(|\tau - \tau'|)\varphi(\tau')\, d\tau', \tag{2}$$

we can use classical projection methods where Eq. (1) is replaced by

$$T_n\varphi_n - z\varphi_n = f, \tag{3}$$

T_n being a Galerkin, Sloan (iterated Galerkin), Kantorovitch or Kulkarni approximation of T (see [4]).

In the examples to be shown, kernel g can be either the $-log(s/2), s \in]0, 2]$ kernel (see [3]) or the radiative transfer in stellar atmospheres kernel, as described in [1] .

For the numerical solution of (3), using projection methods, the evaluation of a discretization matrix A_n, which represents the integral operator T_n restricted to a finite dimensional space X_n, is required.

The precision of the approximate solution depends, not only on the projection method used, but also on dimension of the discretization subspace, on the basis of this subspace, and on the precision of the evaluation of this discretization matrix.

The choice of the basis must take in account the properties of the space where the problem is set, and the discontinuities of the kernel and of the source term f.

For instance, for the second problem mentioned, [1] shows that X should be the Banach space L^1 and the basis of X_n as simple as possible, but based on a grid that can include the discontinuities. In [2] and [3] some relations between the basis and the error on the solutions are shown for some projection methods. Here we will discuss other cases.

This is a joint work with Rosário Fernandes, from the University of Minho, Maria João Rodrigues, from the University of Porto, and Mario Ahues, from the University of Saint Etienne.

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