A FAST FOURIER–GALERKIN METHOD SOLVING A BOUNDARY INTEGRAL EQUATION FOR THE BIHARMONIC EQUATION

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We develop a fast Fourier–Galerkin method for solving a boundary integral equation which is a reformulation of the Dirichlet problem of the biharmonic equation. The proposed method is based on a splitting of the resulting boundary integral operator. That is, we write the operator as a sum of two integral operators, one having the Fourier basis functions as eigenfunctions and the other whose representation matrix in the Fourier basis can be compressed to a sparse matrix having only $\mathcal{O}(n \log n)$ number of nonzero entries, where n is the order of the Fourier basis functions used in the method. We then project the solution of the boundary integral equation onto the space spanned by the Fourier basis functions. This leads to a system of linear equations. A fast solver for the system is based on a compression of its coefficient matrix. We show that the method has the optimal convergence order $\mathcal{O}(n^{-q})$, where q denotes the degree of regularity of the exact solution of the boundary integral equation and requires computing only $\mathcal{O}(n \log n)$ number of entries of the coefficient matrix. Numerical examples are presented to confirm the theoretical results for the approximation accuracy and computational complexity of the proposed method.