Abstracts of IWANASP, September 10 – 12, 2008, Ericeira, Portugal

CONTINUOUS AND DISCRETE GENERALIZATIONS OF HALANAY'S INEQUALITY

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The original version of Halanay's inequality was applied by Halanay [1] to establish exponential stability of solutions of a delay differential equation with a bounded lag such as

$$y'(t) = Ay(t) + By(\alpha(t)) \quad (\alpha(t) = t - \tau \text{ with } \tau > 0) \quad \text{for } t \in [t_0, \infty), \tag{1}$$

under τ-independent conditions. Subsequently, discrete analogues have been established.

There is now a considerable literature, some of it overlapping, related to Halanay's inequality. In this talk we reassess existing results (notably, some derived with my former PhD student Arsalang Tang), place the results in context, and generalise Halanay's original results by establishing a theory on asymptotic stability for solutions of *nonlinear* extensions of (1) with the condition on the delayed argument $\alpha(t)$ (compare (1)) relaxed, for example where

$$\alpha(t) = t - \tau(t), \quad \tau(t) > 0 \text{ for } t \in [t_0, \infty), \quad \text{and } \alpha(t) \to \infty \text{ as } t \to \infty. \tag{2}$$

It follows that the *time lag* $\tau(t)$ *in (2) can be unbounded.* Multiple delay terms are also admitted along with continuously distributed delay terms – giving rise to Volterra integro-differential equations – and we give corresponding results for some discretized versions of such equations. Converse theorems are also indicated.

Past discussions with Dr Arsalang Tang and with Dr Evelyn Buckwar are acknowledged with thanks.

REFERENCES

[1] A. Halanay. *Differential equations. Stability, oscillations, time lags.* Academic Press, New York & London, 1966.