Abstracts of IWANASP, September 10 – 12, 2008, Ericeira, Portugal

REMARKS ON FOURIER COEFFICIENTS AND OSCILLATORY INTEGRALS

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A mid-nineteenth century result, used by Poisson in his proof of the Euler-Maclaurin summation formula (also an asymptotic expansion) is what is now known as the Fourier coefficient asymptotic expansion. (FCAE). In modern terms, this may be stated as follows: When $f(x)$ is analytic in a region containing $[a, b]$, we have

$$\begin{aligned} \int_a^b f(x)e^{ikx}dx \quad = \quad & -e^{ikb}\left\{\frac{i}{k}f(b) + \frac{i^2}{k^2}f'(b) + \cdots + \frac{i^p}{k^p}f^{(p-1)}(b)\right\} \\ & +e^{ika}\left\{\frac{i}{k}f(a) + \frac{i^2}{k^2}f'(a) + \cdots + \frac{i^p}{k^p}f^{(p-1)}(a)\right\} + \frac{i^p}{k^p}\int_a^b f^{(p)}(x)e^{ikx}dx. \end{aligned} \tag{1}$$

This is trivial to prove and can be useful for evaluating a Fourier coefficient for large values of k. In this talk, I *discuss* extensions of this simple formula to one valid for:
(1) an integrand having an algebraic singularity at an end-point:

$$I_1 = \int_a^b f(x)(x-a)^\alpha \exp[ikx]dx;$$

(2) an *oscillatory* integral

$$I_2 = \int_a^b f(x)\exp[ikg(x)]dx;$$

where both f and g are analytic in $[a, b]$.