# Canonical Functional (Holomorphic) Quantization of Pseudo-Photons in Planar Systems

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|---|---|
| $\texttt{hep-th}/0609239$ | 1. Maxwell Action |
| $\texttt{hep-th}/0703193$ | 2. Extended $U_e(1) \times U_g(1)$ Electromagnetism |
| $\texttt{hep-th}/0703194$ | 3. Functional Quantization |

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... derived phenomenologically and unified in 1861.

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$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad , \quad J_e^\nu = (\rho_e, \mathbf{j}_e)$$

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Equations of Motion: $\dfrac{\delta S}{\delta A_\mu} = 0$

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Regularity of Gauge Fields $\Rightarrow$ Bianchi Identities

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Usual definitions: $E^i = F^{0i}$ , $B^i = \frac{1}{2}\epsilon^{ijk}F_{jk}$

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$\Rightarrow$ Hence fails to describe the full Maxwell Equations at variational level

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either the Dirac string of Wu-Yang fiber-bundle

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– although magnetic monopoles are the only justification for quantization of electric charge have so far not been detected

– however non-regular external electromagnetic fields are common in many physical systems and there is plenty of experimental evidence of violation of the Bianchi Identities

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$$eg = 2\pi\,\hbar\,n$$

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$$\tilde{A}: \ \tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\lambda\rho}F_{\lambda\rho}$$

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EOM $A$ : $\partial_\mu\left(F^{\mu\nu} - \frac{1}{2}\epsilon^{\mu\nu\lambda\rho}G_{\lambda\rho}\right) = J_e^\nu \Leftrightarrow \begin{cases} \nabla\cdot\mathbf{E} = \rho_e \\ \nabla\times\mathbf{B} - \dot{\mathbf{E}} = \mathbf{j}_e \end{cases}$

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$$+ \int d^4x \left[e\left(A_\mu - \tilde{C}_\mu\right)J_e^\mu + g\left(A_\mu - \tilde{C}_\mu\right)J_g^\mu\right]$$

EOM $A$ : $\partial_\mu\left(F^{\mu\nu} - \frac{1}{2}\epsilon^{\mu\nu\lambda\rho}G_{\lambda\rho}\right) = J_e^\nu \Leftrightarrow \begin{cases} \nabla\cdot\mathbf{E} = \rho_e \\ \nabla\times\mathbf{B} - \dot{\mathbf{E}} = \mathbf{j}_e \end{cases}$

EOM $C$ : $\partial_\mu\left(G^{\mu\nu} + \frac{1}{2}\epsilon^{\mu\nu\lambda\rho}F_{\lambda\rho}\right) = J_g^\nu \Leftrightarrow \begin{cases} \nabla\cdot\mathbf{B} = \rho_g \\ \nabla\times\mathbf{E} + \dot{\mathbf{B}} = \mathbf{j}_g \end{cases}$

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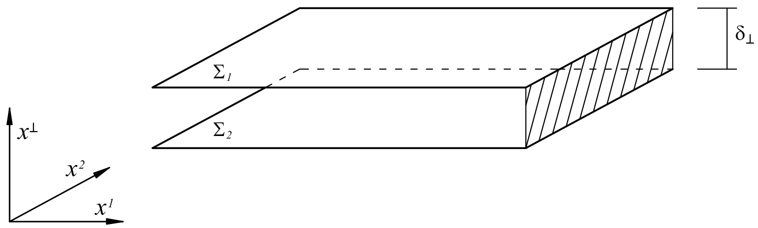

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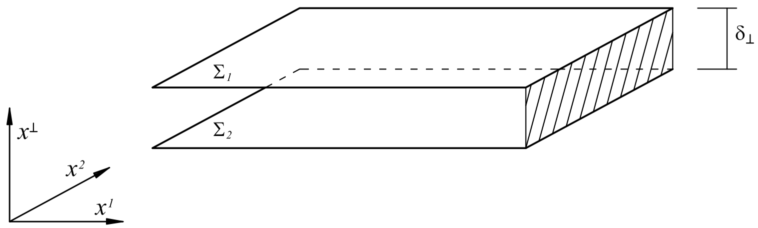


$$I, J = \mu, \perp \quad ; \quad \mu = 0, i, j \quad ; \quad i, j = 1, 2$$

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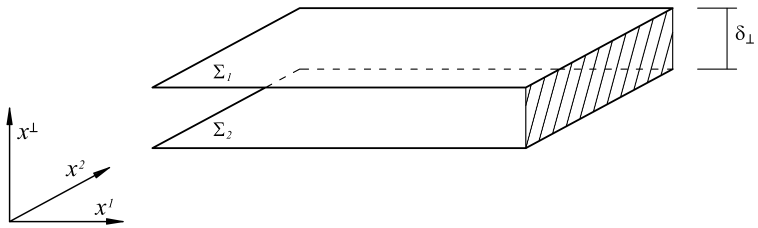


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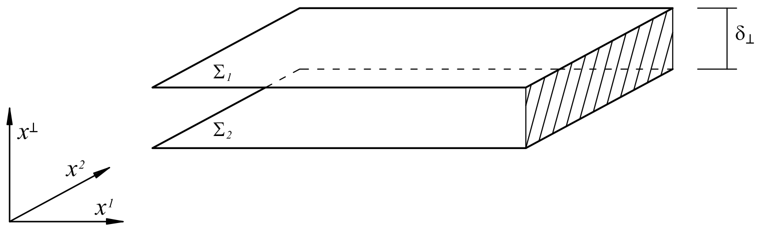


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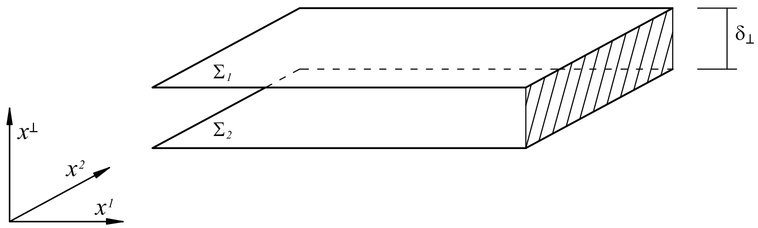


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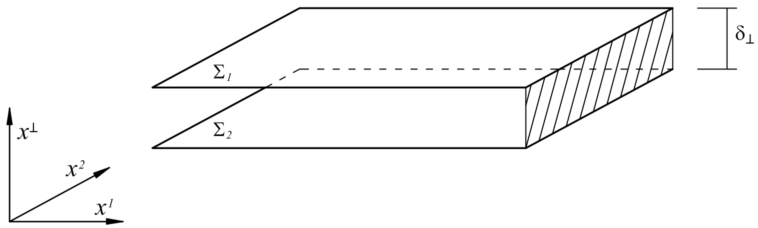


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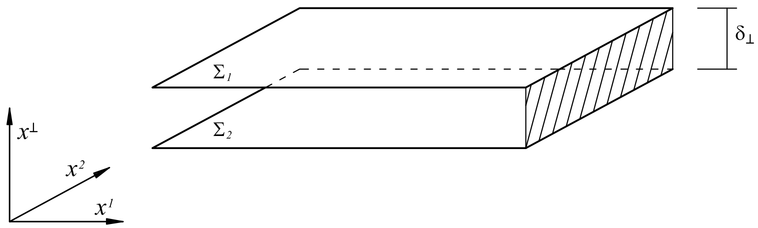


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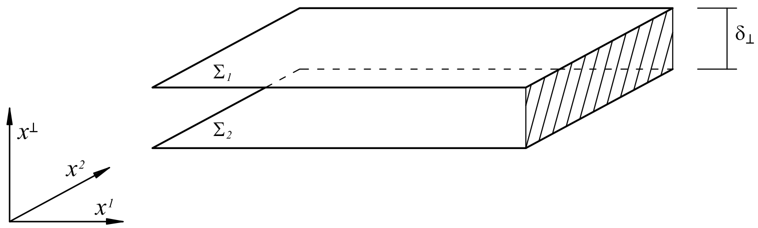


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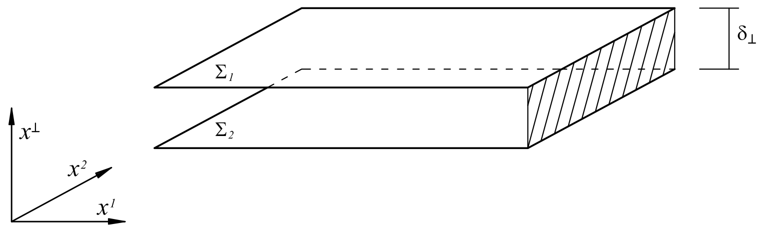


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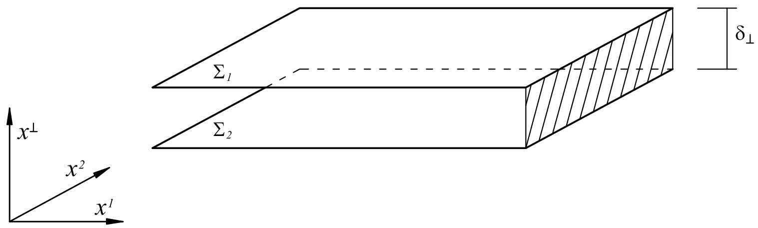


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Id. of bound.: $\displaystyle\int_{\Sigma_1 - \Sigma_2} \epsilon^{\mu\nu\lambda} A_\mu \partial_\nu C_\lambda \equiv \frac{k}{2} \int_{\Sigma} \epsilon^{\mu\nu\lambda} A_\mu \partial_\nu C_\lambda$

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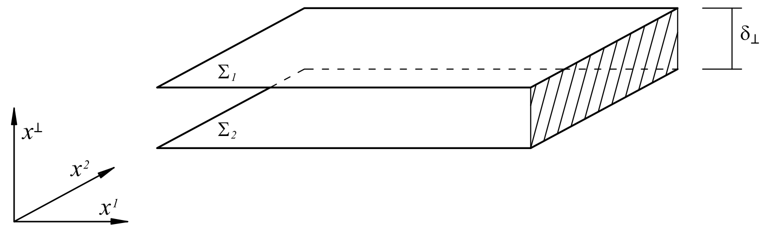

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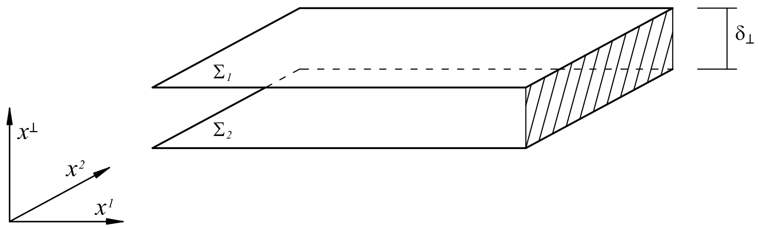


$$\mathcal{L}_4 = -\frac{1}{4} F_{IJ} F^{IJ} + \frac{1}{4} G_{IJ} G^{IJ} + \frac{1}{4} \epsilon^{IJKL} F_{IJ} G_{KL} + A_I \, J_e^I$$

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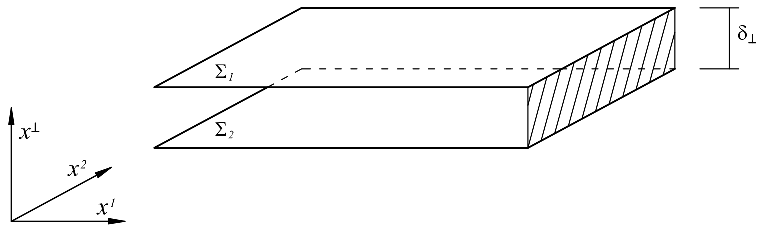


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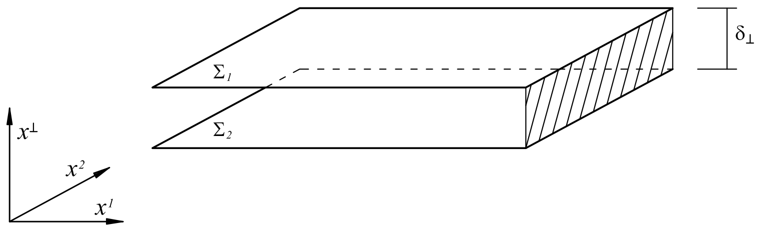


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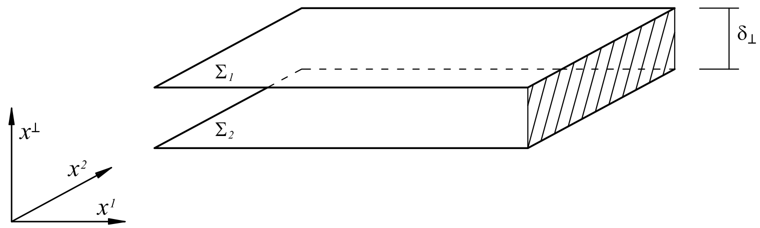


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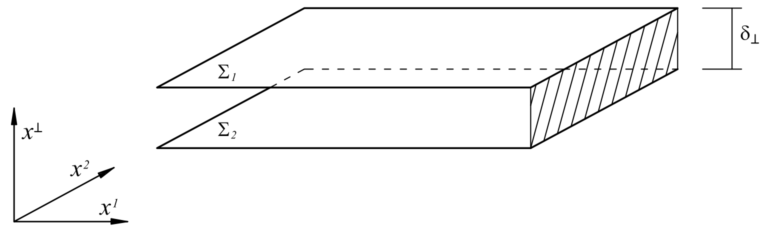


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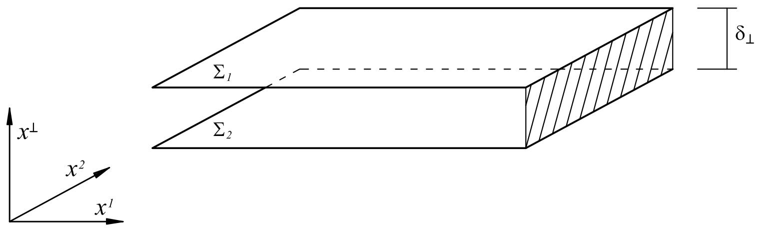
$$S = \int dx^3 \int dx^\perp \mathcal{L}_4 = \int dx^3 \mathcal{L}_3$$

# 2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – Dimensional Reduction

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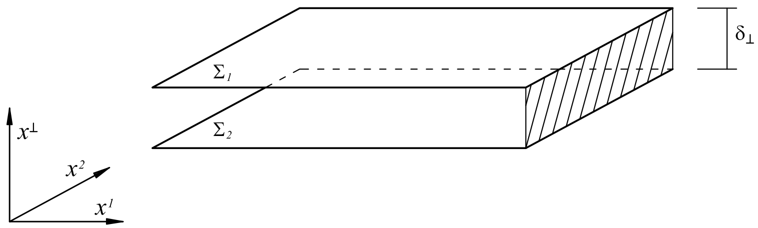


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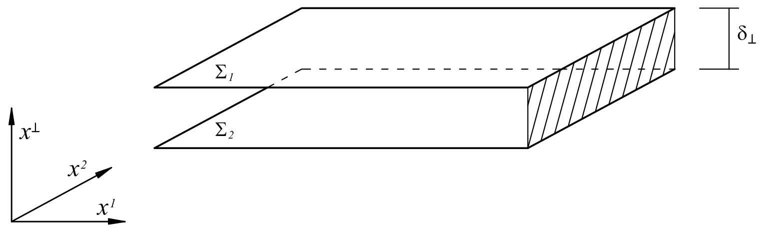


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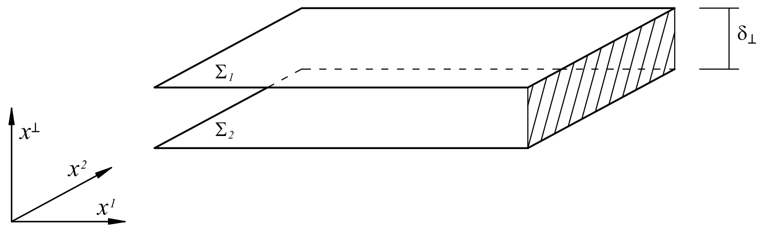


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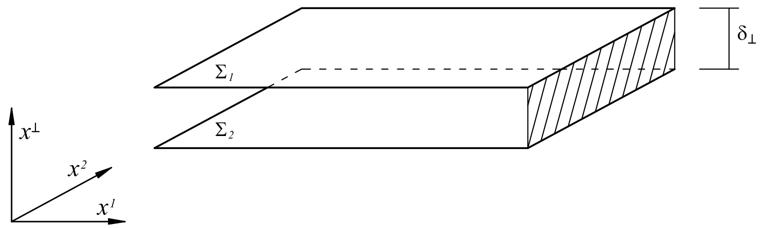


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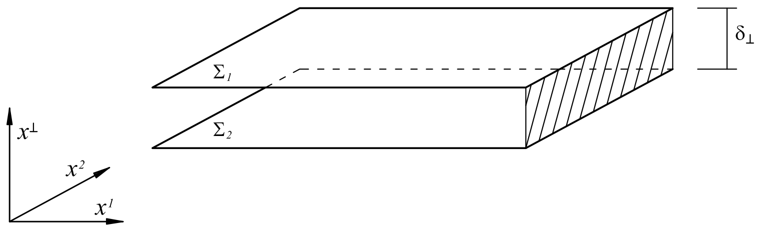


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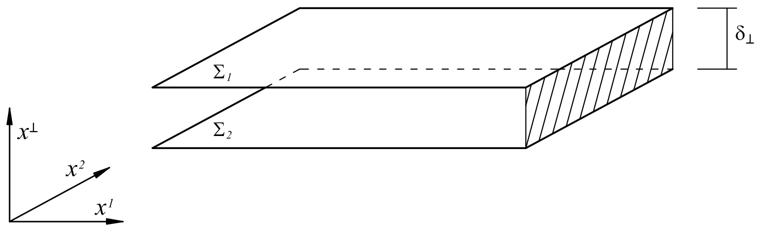


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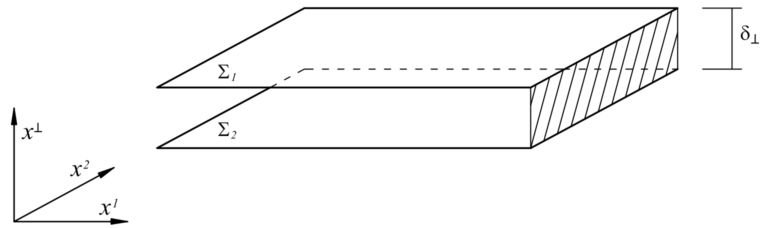


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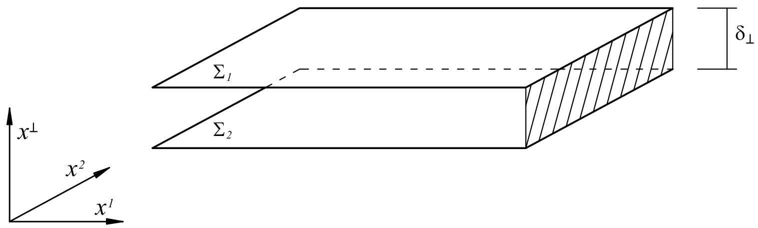


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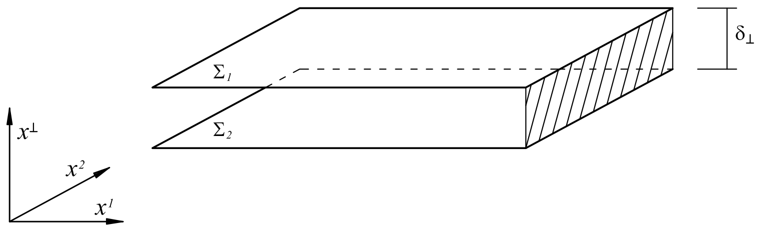


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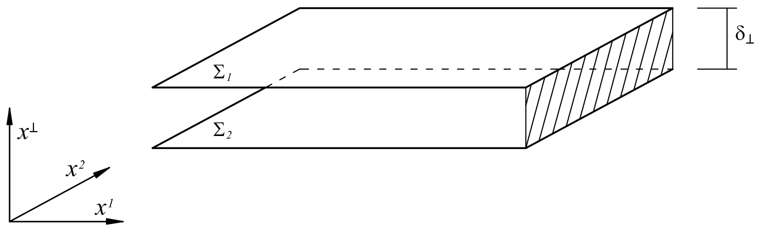


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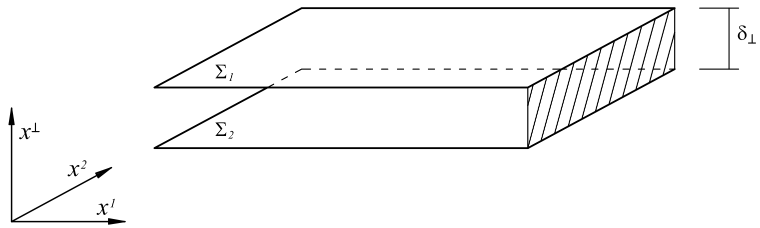


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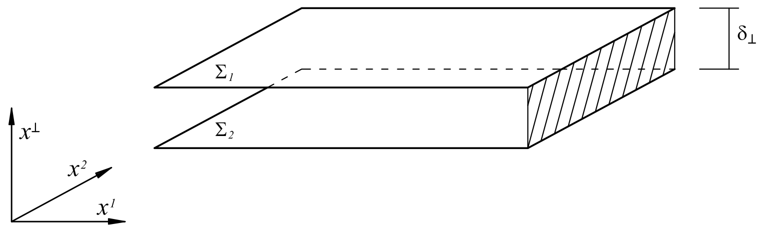


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- However at variational level we have a new theory which solves the inconsistency concerning the Bianchi Identities.
- In planar systems we will obtain new observable consequences when considering Extended $U_e(1) \times U_g(1)$ Electromagnetism.

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- theoretical justification for the low energy contribution to Laughlin's wave function solutions due to the negative energy contributions of pseudo-photon excitations (which are ghost or phantoms).

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- theoretical justification for the orthogonal electric potential due to pseudo-photon electric vortexes which may justify the experimental existence of BEC condensates in bi-layer electron-electron Hall systems instead of its existence in electron-hole Hall systems as originally expected.

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Defining Hamiltonian as usual: $\mathcal{H}_{AC} = +\pi_A^i\partial_0 A_i + \pi_C^i\partial_0 C_i - \mathcal{L}_3$

and considering functional Schrödinger representation:

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we obtain 3 functional constraints:

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$$\Psi_A[C] = \left(1 - i\frac{k}{4\delta_\perp}\epsilon^{ij} A_i^{\rm ext} C_j + \dots\right)\Phi_0[C]$$

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But that is another story...