



International Conference on  
Difference Equations and Applications

ABSTRACTS BOOK  
PROGRAM





International Conference on Difference Equations

## CONTENTS

Introduction .....	1
Main Talks .....	3
Contributed Talks .....	17
List of Participants .....	93
Scientific Program .....	105



# ICDEA 15

Estoril, October 19-October 23

This is the fifteenth International Conference on Difference Equations and Applications (ICDEA 15). The organizing committee gives the welcome to all participants. We hope that the conference be productive and enjoyable for all of you.

Under the auspices of the International Society of Difference Equations (ISDE), the conference is organized by the Center of Mathematical Analysis Geometry and Dynamical Systems of Instituto Superior Técnico (IST), Technical University of Lisbon, and will be held at the hotel Praia-Mar, Carcavelos, Estoril coast.

The purpose of the Conference is to bring together both experts and novices in the theory and application of Difference Equations and Discrete Dynamical Systems. The main theme of the meeting will be “Discrete Dynamical Systems and Nonlinear Science”.

We hope to maintain the scientific framework of the previous conferences held in Instambul (2008), Lisbon (2007), Kyoto (2006), Munich (2005), Los Angeles (2004), Brno (2003), Changsha (2002), Augsburg (2001), Temuco (2000), Poznan (1998), Taipei (1997), Veszprem (1995), and San Antonio (1994).

The Conference will have eleven plenary talks from several areas of Difference Equations, broadly defined, and Discrete Dynamical Systems and their interplay with nonlinear science. The organizers wish to mention their gratitude to all the main speakers for having accepted their invitation, namely to Luís Barreira, Tomasz Downarowicz, Saber Elaydi, Bernold Fiedler, Thorsten Hüls, Sergiy Kolyada, Gerry Ladas, Michał Misiurewicz, Sebastien Schreiber, Sebastian van Strien and James Yorke.

There will be around sixty contributed talks in several areas of Difference Equations and Discrete Dynamical Systems. The conference will publish refereed proceedings of accepted papers. The Proceedings will be published in a good editorial company.

## **International Scientific Committee**

Lluís Alsedà (chair), Jim Cushing, Al Peterson, Alberto Pinto, Carlos Rocha.

## Acknowledgements

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- Center of Mathematical Analysis, Geometry and Dynamical Systems

The organizing committee wishes to express its deep appreciation for the support from the above institutions and also to the members of scientific committee.

A special thanks to Sandra Pereira for her efficient administrative support.

The Organizing Committee

Henrique Oliveira (co-chair)  
José Manuel Ferreira (co-chair)  
Ricardo Coutinho  
Sara Fernandes  
Stefan Hilger  
Rafael Luís  
Carlos Rocha  
Luís Silva  
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**FCT** Fundação para a Ciência e a Tecnologia

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# MAIN TALKS



## Quantitative Poincaré Recurrence

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Poincaré's recurrence theorem is a fundamental result, but only of qualitative nature. This drawback was partly surpassed with the ergodic theorem, which however considers only one quantitative aspect. More recently, there has been a growing interest in the area, starting with the works of Boshernitzan, Ornstein and Weiss. My main aim is to present a sharp description of quantitative recurrence in a rather general setting. I will also describe the somewhat unexpected coincidence between the recurrence rate of the dynamics and the pointwise dimension of an invariant measure, among other recent developments. The talk is intended for a general audience of specialists in dynamical systems, although all the necessary notions and results will be recalled along the way.

### References and Literature for Further Reading

L. Barreira, *Dimension and Recurrence in Hyperbolic Dynamics*, Birkhäuser, 2008.

# Classification of types of convergence of monotone nets of real functions

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Let  $\mathcal{F} = \{f_\kappa : \kappa \in K\}$  be a monotone net of nonnegative functions defined on a metric space  $X$ , converging to an everywhere finite limit function  $f$ . We are familiar with the case when this convergence is uniform. Otherwise, we just say it is “not uniform”. In this talk we will provide a fine classification of all the nonuniform cases, as well. To this end we will introduce certain equivalence relation among such nets, called “uniform equivalence”. The classes of abstraction with respect to this relation correspond to “types of convergence” or “types of defects of uniformity” if one prefers. For example, all nets converging uniformly to a common limit function belong to the same uniform equivalence class.

Further, we will present a number of invariants of the uniform equivalence relation. Among others, we introduce the notion of “order of accumulation” of a net. We will provide theorems and examples which bind this notion with the usual order of accumulation of points in a topological space.

Special role is played by the difference functions  $f - f_\kappa$  or  $f_\iota - f_\kappa$ , where  $\iota > \kappa$ . This is where our topic fits to the scope of the conference. But rather than solving difference equations, we will find solution to a special semicontinuity condition imposed upon these differences, and called “superenvelope”. In other words, we will introduce the notion of an “asymptotically upper semi-continuous net”, and we will say how an arbitrary net can be “fixed”, i.e., turned into an asymptotically upper semi-continuous one, by adding one “correction function” to all its members.

The study of all above objects is motivated by the theory of entropy structures in topological dynamical systems. This application, however, will be outlined only very briefly, as the focus of the lecture is on the abstract subject, as in the title.

## Nonautonomous competition models

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In this talk we will investigate the stability and bifurcation of two dimensional periodic competition models. Moreover, the question of attenuation and resonance will be discussed.

## Noninvasive control by delayed feedback

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We attempt noninvasive stabilization of unstable periodic phenomena  $x(t)$  with minimal period  $p$ . Following an idea of Pyragas, our feedback only involves differences  $x(t) - x(t - \tau)$ . When the time delay  $\tau$  is chosen to be an integer multiple  $np$  of the minimal period  $p$ , the difference and the feedback vanish alike: the control strategy becomes noninvasive on the target periodic orbit.

We discuss the so-called "odd number limitation" which claims that periodic orbits cannot be stabilized, if they possess an odd number of positive real unstable Lyapunov exponents. We present specific control strategies for subcritical Hopf bifurcation and saddle node bifurcations, including model simulations and experiments for a tandem laser system. After an excursion into simple spatio-temporal pattern selection for coupled oscillators, we conclude with a caveat on a "Floquet limitation", beyond which delayed feedback control fails.

The results are joint work with V. Flunkert, M. Georgi, P. Hövel, W. Just, E. Schöll, H.-J. Wünsche, and S. Yanchuk.

## On $r$ -periodic orbits of $k$ -periodic maps

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In this talk, we analyze  $r$ -periodic orbits of  $k$ -periodic difference equations, i.e.

$$x_{n+1} = F_n(x_n), \quad F_n = F_{n \bmod k}, \quad x_n = x_{n \bmod r}, \quad n \in \mathbb{N}$$

and their stability. These special orbits were introduced in [2]. We discuss that, depending on the values of  $r$  and  $k$ , such orbits generically only occur in finite dimensional systems that depend on sufficiently many parameters, i.e. they have a large codimension in the sense of bifurcation theory. As an example, we consider the periodically forced Beverton-Holt model, for which explicit formulas for the globally attracting periodic orbit, having the minimal period  $k = r$ , can be derived. When  $r$  factors  $k$  the Beverton-Holt model with two time-variant parameters is an example that can be studied explicitly and that exhibits globally attracting  $r$ -periodic orbits. For arbitrarily chosen periods  $r$  and  $k$ , we develop an algorithm for the numerical approximation of an  $r$ -periodic orbit and of the associated parameter set, for which this orbit exists. We apply the algorithm to the generalized Beverton-Holt and another example that exhibits periodic orbits with  $r$  and  $k$  relatively prime.

### References

- [1] W.-J. Beyn, T. Hüls, M.-C. Samtenschnieder. On  $r$ -periodic orbits of  $k$ -periodic maps. *J. Difference Equ. Appl.*, 14(8):865–887, 2008.
- [2] S. Elaydi and R. J. Sacker. Global stability of periodic orbits of non-autonomous difference equations and population biology. *J. Differential Equations*, 208(1):258–273, 2005.

## Topology of minimal sets

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Discrete dynamical systems given by a continuous map on a topological (usually compact metrizable) space will be considered. Minimality of such a system/map can be defined as the density of all forward orbits. Every compact system contains at least one minimal set, i.e., a nonempty closed invariant subset such that the restriction of the map to this subset is minimal. A fundamental question in topological dynamics is the one on the topological structure of minimal sets of continuous maps in a given space (and, in particular, whether the space itself admits a minimal map or not). In the first part of the talk we will present a survey of some known facts on minimality (including those on topological properties of minimal maps and on noninvertible minimal maps). Then we will concentrate on the topological structure of minimal sets.

Among others, recent results obtained jointly with Lubomir Snoha and Sergey Trofimchuk will be presented.

# Open Problems and Conjectures in Difference Equations

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We present some open problems and conjectures about some interesting types of rational difference equations and systems of rational difference equations. We are primarily interested in the boundedness nature of solutions, the periodic character of the equations, the global stability behavior of the equilibrium points, and with convergence to periodic solutions including periodic trichotomies.

## References and Literature for Further Reading

- [1] A.M. Amleh, E. Camouzis, G. Ladas, and M. Radin Patterns of Boundedness of a Rational System in the Plane, *J. Difference Equ. Appl.*, (2010).
- [2] E. Camouzis, M. R. S. Kulenović, G. Ladas, and O. Merino, *Rational Systems in the Plane*, *J. Difference Equ. Appl.*, **15** (2009), 303-323.
- [3] E. Camouzis and G. Ladas, *Dynamics of Third-Order Rational Difference Equations; With Open Problems and Conjectures*, Chapman & Hall/CRC Press, (2008).
- [4] E. Camouzis and G. Ladas, Global results on rational systems in the plane, I, *J. Difference. Equ. Appl.*, (2009).
- [5] J. M. Cushing, S. Leverage, N. Chitnis and S. M. Henson, Some discrete competition models and the competitive exclusion principle , *J. Difference Equ. Appl.* 10(2004), 1139-1152.
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## Rectangular braids

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A refinement of the Sharkovsky Theorem for interval maps tells us that there is a forcing relation on cyclic permutations: if a continuous map of an interval has a periodic orbit with a given permutation  $A$  (we look at which point is mapped to which) and  $A$  forces  $B$ , then this map has a periodic orbit with permutation  $B$ . A similar theory exists for orientation preserving disk homeomorphisms, but instead of permutations, one looks at the braid types of the periodic orbits (mapping classes of the disk punctured at the points of the orbit). However, the braid type depends on the behavior of the map off the orbit. We find a class of square homeomorphisms for which all the information about braid types of periodic orbits is encoded in a pair of permutations: vertical order vs. horizontal order, and temporal order vs. horizontal order. We find some simple invariants for braid types of those orbits.

This is a joint research with Ana Rodrigues.

## Dispersal mediated persistence in heterogeneous environments

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Ecologists have long recognized that movement of individuals across space can mediate persistence of interacting populations. In this talk, I will explore this theme using empirically motivated variations of two classical models: Lewontin & Cohen's model of a population living in a temporally variable environment and May & Leonard's model of rock-paper-scissor dynamics. For both models, I will illustrate how interacting populations can persist regionally at intermediate dispersal rates despite being extinction prone locally. The proofs underlying these examples are based on a developing mathematical theory of persistence for random and deterministic models of interacting populations.

### References and Literature for Further Reading

- [1] M. Benaïm and S.J. Schreiber, Persistence of structured populations in random environments, *Theoretical Population Biology* Vol. 76 (2009), pp. 19-34.
- [2] J Hofbauer and S.J. Schreiber, Robust permanence of interacting structured populations, *Preprint* (2009)
- [3] S.J. Schreiber, Interactive effects of dispersal, spatial heterogeneity, and temporal correlations on metapopulation persistence, *Preprint* (2009)
- [4] S. Kirkland, C.K. Li, and S.J. Schreiber, On the evolution of dispersal in patchy landscapes, *SIAM Journal on Applied Mathematics* Vol. 66 (2006), pp. 1366-1382

## Iterations of smooth interval maps, 30 years beyond

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In this talk I will survey how the study of iterations of smooth interval maps has progressed during the last 30 years. The survey will start with describing what was known about iterations of quadratic maps 30 years ago, and what tools have been introduced since. It will review what is now known (and unknown) about iterations of these and more general real analytic interval maps. We will discuss renormalisation theory, monotonicity of topological entropy, density of hyperbolic maps, ergodic properties of maps and the Palis conjecture. The talk will be non-technical.

This talk will be the basis of a survey paper, and also for a new version of a book.

### References and Literature for Further Reading

- [1] S. van Strien, Iterations of smooth interval maps, 30 years beyond, *In preparation*
- [2] W. de Melo and S. van Strien, One Dimensional Dynamics. Springer-Verlag, 1993. Updated version in preparation

## Infinitely Many Cascades Must Exist as Chaos Arises in Dimension 2

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Evelyn Sander (of George Mason University) and I have established a theory of why period-doubling cascades exist, including why systems have infinitely many cascades. Feigenbaum's results describe how a cascade's bifurcations scale – if the cascade exists. We show that infinitely many cascades must exist as a system goes from having only finitely many periodic orbits to chaotic dynamics. Our theory is for generic smooth one-parameter maps  $F(a, x)$  where  $x$  is  $n$ -dimensional but here  $n = 2$ . Below is one of our results for maps with horseshoes in dimension 2, such as the time-1 map of the forced damped pendulum or double-well Duffing equation (where the forcing period is 1).

**Theorem. The Route to Chaos Theorem.** Assume  $F(a, x)$  is smooth and  $x$  is two-dimensional. Under additional mild restrictions, if there are parameter values  $a_0$  and  $a_1$  for which:

$F(a_0, \cdot)$  has at most a finite number of periodic orbits, and

$F(a_1, \cdot)$  has chaotic dynamics and has at most finitely many attractor and repeller periodic orbits.

Then there are infinitely many period-doubling cascades between  $a_0$  and  $a_1$ .

The definition we use for “chaotic dynamics” is satisfied if the map has a homoclinic point. The above result also holds when  $x$  is one-dimensional with minor wording changes. For a preprint, see <http://arxiv.org/abs/0903.3613>



# CONTRIBUTED TALKS



## A linear non autonomous area-preserving map associated to the Szegő recurrence

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Orthogonal sequences  $\{\tilde{P}_n(z)\}_{n=0}^{\infty}$  of monic polynomials on the unit circle,  $\partial\mathbb{D} = \{z \in \mathbb{C} : |z| = 1\}$ , verify the Szegő forward recurrence relation,  $\tilde{P}_0(z) = \tilde{P}_0^*(z) = 1$ ,

$$\begin{pmatrix} \tilde{P}_n(z) \\ \tilde{P}_n^*(z) \end{pmatrix} = \begin{bmatrix} z & \delta_n \\ \bar{\delta}_n z & 1 \end{bmatrix} \begin{pmatrix} \tilde{P}_{n-1}(z) \\ \tilde{P}_{n-1}^*(z) \end{pmatrix}.$$

The  $\delta_n = \tilde{P}_n(0)$  are the reflection coefficients and  $\tilde{P}_n^*(z)$  are the reversed polynomials. We introduce the following change, with  $\|\tilde{P}_n(z)\| = \prod_{j=1}^n \sqrt{1 - |\delta_j|^2}$ , which transforms the original recurrence in a map on the unit sphere of the algebra of coquaternions.

$$\begin{pmatrix} \tilde{P}_n(z) \\ \tilde{P}_n^*(z) \end{pmatrix} = e^{i\frac{n\theta}{2}} \|\tilde{P}_n(z)\| \begin{pmatrix} \hat{P}_n(z) \\ \hat{P}_n(z) \end{pmatrix}.$$

This unit sphere forms a non-compact group isomorphic to  $SL(2; \mathbb{R})$ . It allows us to introduce a linear area-preserving map  $\mathbf{x}_{n+1} = \mathbf{L}_n \mathbf{x}_n$ ; where vectors  $\mathbf{x}_n = (x_1(n); x_2(n)) \in \mathbb{R}^2$ . Companion factorization of matrices  $\mathbf{L}_n \in SL(2; \mathbb{R})$  comes out its associated second order linear homogeneous difference equation, with variable coefficients. It provides an explicit representation of the solutions and simplifies the analysis of some features of Szegő recurrence.

It is a joint work with Mustapha Rachidi, Académie de Reims, France.

### References and Literature for Further Reading

- [1] G. Szegő, *Orthogonal Polynomials*, Colloq. Pub., 23, Amer. Math. Soc., Providence, Rhode Island, (fourth edition), (1975).
- [2] S. Ivanov and S. Zamkovoy, Para-Hermitian and Para-Quaternionic manifolds, *Differential Geom. Appl.* 23 (2005), no. 2, 205-234.

## Tilings and Anosov Diffeomorphisms

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A. Pinto and D. Sullivan [4] proved a one-to-one correspondence between: (i)  $C^{1+}$  conjugacy classes of expanding circle maps; (ii) solenoid functions and (iii) Pinto-Sullivan's dyadic tilings on the real line.

A. Pinto [1,3] introduced the notion of *golden tilings* and proved a one-to-one correspondence between (i) smooth conjugacy classes of Anosov diffeomorphisms, with an invariant measure absolutely continuous with respect to the Lebesgue measure, that are topologically conjugate to the linear automorphism  $G(x, y) = (x + y, x)$ , (ii) affine classes of golden tilings and (iii) solenoid functions.

Here we extend this last result and we exhibit a natural one-to-one correspondence between (i) smooth conjugacy classes of Anosov diffeomorphisms, with an invariant measure absolutely continuous with respect to the Lebesgue measure, that are topologically conjugate to the linear automorphism  $G(x, y) = (ax + y, ax)$ , where  $a \in \mathbb{N}$ , (ii) affine classes of tilings in the real line and (iii) solenoid functions. The solenoid functions give a parametrization of the infinite dimensional space consisting of the mathematical objects described in the above equivalences.

### References and Literature for Further Reading

- [1] A. A. Pinto, J. P. Almeida and A. Portela, Golden tilings, *submitted*.
- [2] A. A. Pinto, D. Rand, Renormalisation gives all surface Anosov diffeomorphisms with a smooth invariant measure, *submitted*.
- [3] A. A. Pinto, D. Rand and F. Ferreira, Fine Structures of Hyperbolic Diffeomorphisms, Springer Monographs in Mathematics, Springer Verlag, (2009).
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## A refinement of Sharkovsky's theorem for periodic difference equations

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We discuss a new characterization of periodic solutions of  $p$ -periodic difference equations, and classify the periods into multiples of  $p$  and non-multiples of  $p$ . We show that the elements of the set of multiples of  $p$  follow the well-known Sharkovsky's ordering multiplied by  $p$ . On the other hand, we show that the elements of the set  $\Gamma_p$  of non-multiples of  $p$  are independent in their existence. Based on the proposed classification, we give a refinement of Sharkovsky's theorem for periodic difference equations. Finally, we show the existence of a  $p$ -periodic difference equation with infinite  $\Gamma_p$ -set in which the maps are defined on a compact domain and agree exactly on a set of zero Lebesgue measure.

### References and Literature for Further Reading

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- [2] Z. AlSharawi, Periodic orbits in periodic discrete dynamics, *Computers & Mathematics with Applications*, 56 (2008) 1966–1974
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**Acknowledgment:** This is a collaborative work with Ahmad Al-Salman, Sultan Qaboos University, Oman

## Quadratic dynamics in matrix algebras

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We describe the dynamics in matrix algebras under iteration of quadratic maps, using the algebraic properties of matrix operations. We also discuss the behavior of the attractive cycles and present results involving the non-critical attractive cycles, introduced in [1] and [2].

### References and Literature for Further Reading

- [1] A. K. Serenevy, Dynamics of polynomial maps of 2x2 real matrices, *Topology proceedings* 26 (2001) 763-778.
- [2] A. Baptista, C. Correia Ramos, N. Martins, Difference equations on matrix algebras, *Proceedings of the International Conference on Difference Equations and Applications*, Portugal, 23 - 27 July 2007.

## The structure of kneading curves for Lozi maps

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In [1], René Lozi studied the interesting dynamical behaviour of the map of the plane

$$L(x, y) = (1 - 1.7|x| + y, 0.5y).$$

A few years later, Michal Misiurewicz, [2], characterized the set of parameter values for which the family of maps of the plane

$$L_{ab}(x, y) = (1 - a|x| + y, by)$$

have a strange attractor and found a trapping zone which allowed him to construct the attractor. From the kneading theory presented by Yutaka Ishii in [3], we generalize the concept of critical point and of kneading sequence for a map  $L_{ab}$ , [4]. In this work, we characterize the structure of the kneading curves for the parameter plane region corresponding to mappings with a strange attractor.

### References and Literature for Further Reading

- [1] R. Lozi, *Un attracteur étrange du type attracteur de Hénon*. J. Physique (Paris) **39 (Coll. C5)** (1978), 9-10.
- [2] M. Misiurewicz, *Strange attractors for the Lozi mappings*. Nonlinear Dynamics **R. G. Helleman (ed.)** New York: The New York Academy of Sciences (1980)
- [3] Yutaka Ishii, *Towards a kneading theory for Lozi mappings I: A solution of pruning front conjecture and the first tagency problem*. Nonlinearity **10** (1997)731-747
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## Positive and Oscillating Solutions of Linear Discrete Equations

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The phenomenon of the existence of a positive solution of difference equations is often encountered when analysing mathematical models describing various processes. This is a motivation for an intensive study of the conditions for the existence of positive solutions of difference equations. Such analysis is related to an investigation of the case of all solutions being oscillating. In the talk, conditions for the existence of a positive solution are given for a class of linear delayed discrete equations. For the same class of equations, also conditions are given for all the solutions being oscillating. The results obtained indicate sharp sufficient conditions for the existence of a positive solution or for the case of all solutions being oscillating.

We consider the delayed scalar linear discrete equation of the second order

$$\Delta x(n) = -p(n)x(n-1) \tag{1}$$

where  $n \in \mathbb{Z}_a^\infty := \{a, a+1, \dots\}$ ,  $a \in \mathbb{N}$  is fixed,  $\Delta x(n) = x(n+1) - x(n)$ ,  $p: \mathbb{Z}_a^\infty \rightarrow \mathbb{R}^+ := (0, \infty)$ . A solution  $x: \mathbb{Z}_{a-1}^\infty \rightarrow \mathbb{R}$  of (1) is called positive (negative) if  $x(n) > 0$  ( $x(n) < 0$ ) on  $\mathbb{Z}_{a-1}^\infty$ . The solution  $x = x(n)$  of (1) being neither positive nor negative on  $\mathbb{Z}_{a_1-1}^\infty$  for arbitrary  $a_1 \geq a$  is called oscillating.

**Definition 1.** Let us define the expression  $\ln_q t$ ,  $q \geq 1$ , by  $\ln_q t = \ln(\ln_{q-1} t)$ ,  $\ln_0 t \equiv t$  where  $t > \exp_{q-2} 1$  and  $\exp_s t = \exp(\exp_{s-1} t)$ ,  $s \geq 1$ ,  $\exp_0 t \equiv t$  and  $\exp_{-1} t \equiv 0$  (instead of  $\ln_0 t$ ,  $\ln_1 t$ , we will only write  $t$  and  $\ln t$ ).

In [1] a delayed linear difference equation of higher order is considered and the following result related to equation (1) on existence of a positive solution solution is proved.

**Theorem 1.** Let  $a \in \mathbb{N}$  and  $q \in \mathbb{N}$  be fixed. Suppose that

$$p(n) \leq \frac{1}{4} + \frac{1}{16n^2} + \frac{1}{16(n \ln n)^2} + \frac{1}{16(n \ln n \ln_2 n)^2} + \frac{1}{16(n \ln n \ln_2 n \ln_3 n)^2} \\ + \dots + \frac{1}{16(n \ln n \ln_2 n \dots \ln_q n)^2}. \quad (2)$$

for every  $n \in \mathbb{Z}_a^\infty$ . Then there exists a positive integer  $a_1 \geq a$  and a solution  $x = x(n)$ ,  $n \in \mathbb{Z}_{a_1-1}^\infty$  of equation (1) such that  $x(n) > 0$  for every  $n \in \mathbb{Z}_{a_1-1}^\infty$ .

Our goal is to answer an open question whether all solutions of (1) are oscillating if inequality (2) is replaced by an opposite inequality. The proof of following result uses a consequence from one of results of Y. Domshlak [2].

**Theorem 2.** Let  $a \in \mathbb{N}$  be sufficiently large and  $\kappa > 1$ . Suppose that

$$p(n) > \frac{1}{4} + \frac{1}{16n^2} + \frac{1}{16(n \ln n)^2} + \frac{1}{16(n \ln n \ln_2 n)^2} + \frac{1}{16(n \ln n \ln_2 n \ln_3 n)^2} \\ + \dots + \frac{1}{16(n \ln n \ln_2 n \dots \ln_{(q-1)} n)^2} + \frac{\kappa}{16(n \ln n \ln_2 n \dots \ln_q n)^2} \quad (3)$$

for every  $n \in \mathbb{Z}_a^\infty$ . Then all solutions of equation (1) are oscillating.

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## Difference equations and entropy of interval maps

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We study piecewise affine interval maps with countably many laps. For two classes of such maps we investigate (the best) bounds of the entropy. We show that there is a close coherence between these bounds and bifurcation values of parameters determining strictly monotone solutions to special difference equations.

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# Real Dynamics of Integrable Birational Maps

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This talk deals with the dynamic generated by iteration of birational maps in  $\mathbb{R}^k$  with  $k - 1$  independent rational first integrals. Selected a level curve  $\Sigma_c$ , we begin by distinguishing between two algebraic subsets of  $\Sigma_c$ , being one of them the good set for the study of the iterates of the map. After, we prove that this good set can be desingularized and compactified in such a way that the study of  $F$  reduces to the study of some orientation preserving homeomorphisms on circles and intervals.

When the birational map is measure-preserving, we prove that in most cases the preserving homeomorphisms on the corresponding circles are conjugated to rotations.

The results explained in this talk are based on the joint work [1].

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## Markov subshifts and infinite dimension dynamical systems arising from iterated maps

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We consider the dynamical system  $(\mathcal{A}, T)$ , where  $\mathcal{A}$  is a class of smooth real functions defined on some interval and  $T : \mathcal{A} \rightarrow \mathcal{A}$  is an operator  $T\phi := f \circ \phi$ , where  $f$  is a function on the real line. Iteration of smooth maps appears naturally in the study of continuous difference equations and boundary value problems such as those in Romanenko *et al* [2], Severino *et al* [3], Sharkovsky *et al* [4]-[5], Vinagre *et al* [6]. In our work we develop some techniques of symbolic dynamics for the dynamical system  $(\mathcal{A}, T)$ . We analyze in detail the case in which  $T$  arises from a unimodal family of interval maps  $f$ . The obtained dynamical system is infinite dimension, nevertheless if  $f$  has a stable cycle then we can build a countable Markov partition for  $(\mathcal{A}, T)$  which characterizes the overall behavior of the system, namely its asymptotic behavior.

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## Coexistence of uncountably many attracting sets

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We study two-dimensional skew-products defined on the cylinder  $\mathbb{S}^1 \times \mathbb{R}$  of the form:

$$\begin{cases} \theta_{n+1} = R(\theta_n), \\ x_{n+1} = f(x_n)g(\theta_n) \end{cases}$$

where  $R : \mathbb{S}^1 \rightarrow \mathbb{S}^1$  is a continuous degree-one circle map,  $f : \mathbb{R} \rightarrow \mathbb{R}$  satisfies some properties of concavity and monotonicity, and the map  $g : \mathbb{S}^1 \rightarrow \mathbb{R}$  is continuous. We prove that when  $R$  has no periodic points, i.e. its rotation number is irrational, there exists finitely many attracting sets. In contrast, when the rotation interval of any of the lifts of  $R$  is non-degenerate, we prove the existence of uncountably many attracting sets, each of them related with a different irrational rotation number. This work is part of my PhD thesis supervised by Prof. Lluís Alsedà.

## Ważewski's method for systems of dynamic equations on time scales

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The Ważewski's method, which is well-known for ordinary differential equations, is developed for a system of dynamic equations on an arbitrary time scale. Sufficient conditions guaranteeing the existence of at least one solution with graph staying in previously defined open set are derived. This result, generalizing some previous results concerning the asymptotic behavior of solutions of discrete equations, is suitable for investigating of asymptotic behavior of solutions of dynamical systems.

We will assume that a time scale  $\mathbb{T}$  has the following properties:  $\mathbb{T} \subset [t_0, \infty)$ ,  $t_0 \in \mathbb{R}$  and  $t_0 \in \mathbb{T}$ . Let

$$b_i, c_i: \mathbb{T} \rightarrow \mathbb{R}, \quad i = 1, \dots, n$$

be delta differentiable functions such that  $b_i(t) < c_i(t)$  for each  $t \in \mathbb{T}$ . We define (for  $t \in \mathbb{T}$ ) a set

$$\omega(t) := \{u \in \mathbb{R}^n : b_i(t) < u_i < c_i(t), \quad i = 1, \dots, n\}.$$

Let us consider the dynamic system

$$u^\Delta = f(t, u) \quad (4)$$

where  $f: \mathbb{T} \times \mathbb{R}^n \rightarrow \mathbb{R}^n$ .

Our aim is to establish sufficient conditions for the right-hand side of the system (4) guaranteeing the existence of at least one solution of (4) defined on  $\mathbb{T}$  such that  $u(t) \in \omega(t)$  for each  $t \in \mathbb{T}$ . The main result gives a set of relevant sufficient conditions in terms of so-called points of strict egress for the set  $(t, \omega(t))$ ,  $t \in \mathbb{T}$  with respect to the system (4).

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## On the dynamics of Lorenz maps

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The Lorenz map we are interested in is a map  $f$  on the unit interval  $I = [0, 1]$  such that for some  $c \in (0, 1)$  we have

- (i)  $f$  is strictly increasing on  $[0, c)$  and on  $(c, 1]$ ;
- (ii)  $\lim_{x \uparrow c} f(x) = 1, \lim_{x \downarrow c} f(x) = 0$ .

Lorenz map plays an important role in the study of the global dynamics of families of vector fields near homoclinic bifurcations. It is also closely related to circle map and unimodal map. In this talk, I will discuss the following topics on Lorenz maps:

- 1, renormalization and  $\alpha$ -limit sets of expanding Lorenz map;
- 2, renormalization and conjugacy of piecewise linear Lorenz map;
- 3, acip of piecewise linear Lorenz map;
- 4, rotation numbers of Lorenz shift;
- 5, existence of  $C^1$  hyperbolic interval map with "Invisible" acip and positive entropy;
- 6, construction of differentiable interval map with nested sequence of attracting sets.

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# Oscillation Theorems for Symplectic Difference Systems

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We present recent results concerning oscillatory and spectral properties of symplectic difference systems. In particular, we show that the classical oscillation theorem (which relates the number of eigenvalues to the number of focal points) can be extended to symplectic difference systems with general self-adjoint boundary conditions. The presented results have been achieved in the joint research with Werner Kratz (University of Ulm, Germany).

## Stability of chain recurrent points of a continuous map of the interval

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J. Smítal and T. H. Steele [J. Difference Equ. Appl. 15 (2009), 77 – 86] characterize a function  $g \in C(I)$  at which maps  $C$  and  $NW$  are continuous, and put a problem whether the same characterization is true for the map  $CR$ , which takes an  $f \in C(I)$  to its set of chain recurrent points  $CR(f)$ .

We solve this problem by showing that  $CR$  is continuous at  $g \in C(I)$  if and only if the periodic orbits of  $g$  which are  $p$ -stable, i.e. stable with respect to small perturbations of  $g$ , are dense in the set  $CR(g)$  of chain recurrent points of  $g$ .

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# Comparison theorems for Symplectic difference systems

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We derive new relations between the number of focal points for conjoined bases of symplectic difference systems

$$Y_{i+1} = W_i Y_i, W_i^T J W_i = J, \hat{Y}_{i+1} = \hat{W}_i \hat{Y}_i, \hat{W}_i^T J \hat{W}_i = J.$$

We also consider applications of these relations in the spectral theory for symplectic difference systems.

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## Synchronization, coupling and mixing time in Markov maps

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We present a study that shows the direct relationship that exists between the rate of mixing, time of synchronization and coupling of discrete dynamical systems.

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## Bayesian-Nash Equilibria in a Behavior Model

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We introduce, in the literature, a Game Theoretical Model of Planned Behavior in which we propose Bayesian-Nash Equilibria as one, of many, possible mechanisms of transforming human intentions in behavior. This model establishes an analogy between the Theory of Planned Behavior that analyses the decision-making mechanisms of individuals and Game Theory concepts. We study the best strategic individual decision taking in account the collective response and consequently the use of a constant strategy in no saturation situations and splitted strategies in boredom and frustration situations, distinguishing two different concept worlds.

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## Universal fluctuations in stock exchange markets

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We analyze the Dow Jones Industrial Average (DJIA30) and the Standard and Poor's 100 index (S&P100) of the NYSE stock exchange market. Surprisingly, we discover the data collapse of the histograms of the re-scaled DJIA30 daily return positive and negative fluctuations and of the re-scaled S&P100 index daily return positive and negative fluctuations to the universal non-parametric Bramwell-Holdsworth-Pinton (BHP) distribution. The re-scaling parameter is found by minimization of the Kolmogorov-Smirnov statistic and has a similar value for both indexes. Since the BHP probability density function appears in several other dissimilar phenomena, our result reveals a universal feature of the stock exchange market.

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## Patents in New Technologies

A. A. PINTO<sup>1</sup>, B. M. P. M. OLIVEIRA<sup>2</sup> and M. FERREIRA<sup>1</sup>

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We present a new R&D investment function in a Cournot competition model inspired in the logistic equation. We do a full characterization of the associated game and study the short and long term economical effects derived from using this new R&D investment function. In particular, we find the existence of regions with multiple Nash investment equilibria. For low production costs, that can correspond to the production of old technologies, the long term economical effects are not very sensitive to small changes in the efficiency of the R&D programs neither to small changes in the market structure. However, for high production costs, that can correspond to the production of new technologies, the long term economical effects are very sensitive to small changes in the efficiency of the R&D programs and also to small changes in the market structure.

# Calculus of Variations with Discrete Fractional Derivatives

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We introduce a discrete-time fractional calculus of variations on the set  $h\mathbb{Z} = \{hk : k \in \mathbb{Z}\}$ ,  $h > 0$ . First and second order necessary optimality conditions are established. Examples illustrating the use of the new Euler-Lagrange and Legendre type conditions are given. They show that solutions to the considered fractional problems: (i) become the classical discrete-time solutions when the fractional order of the discrete-derivatives is an integer value, (ii) converge to the fractional continuous-time solutions when  $h$  tends to zero. Examples also show that Legendre type condition is useful to eliminate false candidates. .

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## Strongly inverse positive operators for $n$ th order difference equations

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We consider the general  $n$ th order difference equation

$$T_n[K_0, \dots, K_n] x(k) = f(k, x(k), x(k+1), \dots, x(k+n)), \quad k \in I,$$

coupled with the periodic boundary value conditions

$$x(i) = x(N+i), \quad i = 0, \dots, n-1,$$

where the operator  $T_n[K_0, \dots, K_n] : \mathbb{R}^{N+n} \rightarrow \mathbb{R}^N$  is defined by

$$T_n[K_0, \dots, K_n] x(k) \equiv x(k+n) + \sum_{i=0}^n K_i x(k+i), \quad \text{for all } k \in I,$$

and  $f : I \times \mathbb{R}^{n+1} \rightarrow \mathbb{R}$  is a continuous function.

The operator  $T_n[K_0, \dots, K_n]$  is strongly inverse positive which yields that the corresponding Green's function is strictly positive. Then, by using a new fixed point theorem based on the classical Krasnoselskii's cone expansion/contraction theorem, we deduce the existence of at least one positive solution provided that the nonlinear part of the equation satisfies some monotonicity assumptions and the existence of a positive upper solution.

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## Linearized Riccati technique in discrete half-linear oscillation theory

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We consider the half-linear second order difference equation

$$\Delta(r_k \Phi(\Delta x_k)) + c_k \Phi(x_{k+1}) = 0, \quad \Phi(x) := |x|^{p-2}x, \quad p > 1 \quad (*)$$

and we investigate oscillatory properties of (\*) using properties of an associated linear equation. In particular, we study summation characterization of the recessive solution of (\*). The crucial role in our investigation is played by estimates for a certain nonlinear function which appears in the so-called modified Riccati equation. The presented results were achieved in the joint research with Prof. Ondřej Došlý, Brno.



## On the dynamics of some non-autonomous Lyness type recurrences

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In this talk we describe the full dynamics of second order Lyness' difference equations

$$x_{n+2} = \frac{a_n + x_{n+1}}{x_n}, \quad (5)$$

where  $\{a_n\}_n$  is either a 2-periodic or a 3-periodic sequence of positive values and the initial conditions  $x_1, x_2$  are as well positive. More concretely we prove that in both cases the orbits are bounded and persistent. Furthermore, either it is satisfied that:

- (a) The sequence  $\{x_n\}_n$  generated by (5) is periodic and in this case its period is an even number; or
- (b) The sequence  $\{x_n\}_n$  generated by (5) fulfills densely one or two (resp. one, two or three) disjoint intervals of  $\mathbb{R}$  when  $\{a_n\}_n$  is 2-periodic (resp. 3-periodic). Moreover it is possible by algebraic tools to distinguish which is the situation.

The persistence is a consequence of the existence, in both cases, of invariants found by E.J. Janowski, M.R.S. Kulenović and Z. Nurkanović and of a study of the algebraic curves given by them. The description of the sequences essentially follows from the methods introduced in [1]. The distinction between the number of connected components of the adherence of the sequence  $\{x_n\}_n$  can be done by computing a suitable set of resultants.

This talk is based on the forthcoming paper [2].

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## **Calculus of variations on time scales with delta, nabla and directional derivatives**

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The calculus of variations on time scales unifies and extends the continuous, the discrete-time, and the quantum variational calculus. Such unification and extension is, however, not well defined, in the sense that it depends on the specific differentiation tools one is dealing with, e.g., delta or nabla derivatives. In this work we propose a new and more general approach to the calculus of variations that allows to obtain, as particular cases, the delta or nabla results. More precisely, the unification of delta and nabla calculi, which we introduce in this paper, is based on the use of the directional derivative. This new approach gives useful results: we get unified Euler-Lagrange equations and unified treatment to isoperimetric problems. Pareto optimal solutions are also considered. Relations between the different notions are comprehensively presented. We enrich all results with illustrative examples.

## The Pace of Information Diffusion in a Monetary Policy Model

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Sticky-information monetary policy models emerged in order to offer a framework capable of replicating fundamental empirical regularities with a high degree of accuracy, something that the benchmark sticky prices macroeconomic setup failed to deliver. Such inattentiveness framework was built on the assumption that current decisions are mainly based on past expectations about the current state of the economy. However, why contemporaneous decisions should rely on past expectations has remained an open question. In this presentation, we look inside the black box and propose a candidate explanation for the stickiness of information. The main idea is that a process of information diffusion is triggered by the relation between two rational players: the profit maximizing media industry and the private agents, who seek information in order to update prices. The setup allows to evaluate how different degrees of information diffusion impact over the trajectories of nominal and real variables in the economy.

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## Global stability properties of virus dynamics discrete models

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A sufficient condition for the global stability of positive equilibrium points of difference equations, which appears as a model for basic virus dynamics, is obtained by applying the technique of Liapunov functionals.

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# Critical Higher Order Sturm-Liouville Difference Operators

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This is a joint work with Prof. Ondřej Došlý. We study the criticality of  $2n$ -order Sturm-Liouville difference operators

$$L(y)_k := \sum_{\nu=0}^n (-\Delta)^\nu \left( r_k^{[\nu]} \Delta^\nu y_{k-\nu} \right), \quad k \in \mathbb{Z}.$$

Using a Sturmian-type separation theorem for recessive solutions of Hamiltonian systems (see [1]) we show, that arbitrarily small (in a certain sense) negative perturbation of nonnegative critical operator leads to an operator which is no longer nonnegative. This research is motivated by [2], where the concept of critical operators is introduced for operators of second order.

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## Quivers and Difference Equations

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After a short introduction into the idea of a quiver and its representations we will present the “scalar loop” property. Some considerations concerning this property follow. Three examples will illustrate the theory.

## Multiple equilibria in a dynamic North-South model

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We consider the discrete time version of a dynamic two country model of trade proposed by Shimomura (2007), extend it and show the existence of multiple equilibrium paths. In addition we will prove that if a consumable capital is more labor intensive than a pure consumption good, multiple equilibrium paths can arise even under the normality assumption in consumption.

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## **Discrete host-parasitoid models with Allee effects and age structure in the host**

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We present a stage-structured single species population model with Allee effects. The asymptotic dynamics of the model depend on the maximal growth rate of the population as well as on its initial population size. We also investigate two models of host-parasitoid interaction with stage-structure and Allee effects in the host. The parasitoid population may drive the host population to extinction in both models even if the initial host population is beyond the Allee threshold.

## First-order difference equations with nonlinear boundary conditions

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A monotone iterative method is used to show that corresponding difference problems with boundary conditions have extremal solutions in the region bounded by lower and upper solutions. It is important to indicate that the right-hand-side of problems depended on  $r$  delayed arguments. Difference inequalities of such types are also discussed.

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## Stability cone for difference equation

$$x(n) = Ax(n-1) + Bx(n-k)$$

with the commuting matrices  $A, B$

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We have constructed a stability cone, constituted by the system of ovals on the complex plane. By the working with the eigenvalues of matrices  $A, B$  we create complex numbers, such that the matrix equation in the title is asymptotically stable if and only if the numbers are inside the stability cone.

This is joint work with V. Gilyazev and V. Malygina.

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## Entropy, horseshoes and homoclinic trajectories on trees, graphs and dendrites

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It is known that the positiveness of topological entropy, the existence of a horseshoe and the existence of a homoclinic trajectory are mutually equivalent, for interval maps. The aim of the talk is to provide the relations between the properties for continuous maps of trees, graphs and dendrites. We consider three different definitions of a horseshoe and two different definitions of a homoclinic trajectory. All the properties are mutually equivalent for tree maps, whereas not for maps on graphs and dendrites.

## Remark on numerical nightmare phenomenon

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The talk deals with numerical nightmare phenomenon, which is illustrated on IVP of scalar pantograph equation

$$x(t) = ax(t) + bx(\lambda t), \quad 0 < \lambda < 1, x(0) = 1.$$

We utilize the backward Euler method for discretization. In specific cases the numerical solution gives  $x_n = 0$  for all  $n > N > 0$ . But this is in the contradiction with appropriate asymptotic estimates. The Schur theorem can be utilized to explain this discrepancy. Finally, some computations in Maple will be presented to illustrate numerical nightmare phenomenon.

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## Difference and sum operators in the space of strictly monotonic functions

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An algebraic model for linear functional and linear difference equations is developed, which under specialization reproduces a rich core of known results for classical linear functional and difference equations.

New difference and sum operators are defined, which reduce to classical difference and sum operators under specialization.

These difference and sum operators are defined in terms of a strictly monotonic continuous function  $\phi$  and a canonical function  $X$ .

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## Efficient synchronization of one-dimensional chaotic maps by a non-symmetric coupling

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We consider synchronization phenomena of chaotic discrete dynamical systems with unidirectional and bidirectional coupling mechanisms.

We present a systematic study of the nonlinear coupling scheme

$$\begin{cases} x_{n+1} = a - x_n^2 + \rho(x_n - y_n)^2 \\ y_{n+1} = b - y_n^2 + \rho(x_n - y_n)^2 \end{cases}$$

that appears (Isaeva *et al.* [1]) in natural way from the family of analytic complex quadratic maps when we proceed to the decomposition into real and imaginary parts. It is an asymmetric coupling between two real quadratic maps in which we use different values of the control parameters  $a$  and  $b$  chosen in the region of chaos. The nonlinear coupling term is defined by the square of the difference between the variables  $x$  and  $y$  of the subsystems times the coupling strength  $\rho$ . The map obtained by coupling two chaotic quadratic maps exhibits a richer dynamics than the single one, but it is still possible to study its behaviour. We are not aware about any studies of this type of coupling. When not even practical synchronization (in the Kapitaniak sense) is achieved, but the difference between the dynamical variables of the subsystems is bounded, we still can apply to the coupled system a chaos control technique based on the well-known OGY-method (Ott-Grebog-Yorke [2]), the pole-placement control technique, developed by Romeiras *et al.* [3], in order to decrease the difference  $y - x$ .

Moreover, we obtain stable identical and generalized synchronization with some versions of that original coupling, highlighting the absence of symmetry. Two of

them are generalizations promoting the use of different coupling parameters  $\rho_1$  and  $\rho_2$ . By numerical simulations of the coupled chaotic maps, we identify the range of coupling strengths for which is allowed practical synchronization and locally stable asymptotic synchronization. The attractive effect of enough coupling can counterbalance the trend of the individual chaotic trajectories to separate, due the sensitive dependence on initial conditions, and makes them to approach. In case of coupling with two different coupling parameters, the linear stability of the synchronous state is ensured if some relations are guaranteed between the coupling parameters and the initial conditions.

## Nonautonomous periodic Leslie-Gower competition model

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In this talk we study the dynamics of the nonautonomous difference equation

$$\begin{pmatrix} x_{n+1} \\ y_{n+1} \end{pmatrix} = \begin{pmatrix} \frac{K_n a_n x_n}{K_n + (a_n - 1)x_n + c_n y_n} \\ \frac{L_n b_n y_n}{L_n + (b_n - 1)y_n + d_n x_n} \end{pmatrix} = F_n(x_n, y_n), \quad (6)$$

where  $x_n, y_n$  are the population size of species  $x$  and  $y$ , respectively, at time period  $n$ ,  $K_n, L_n > 0$  are the carrying capacities of species  $x$  and  $y$ , respectively,  $a_n, b_n > 0$  are the intrinsic growth rate of species  $x$  and  $y$ , respectively, while  $c_n, d_n \in (0, 1)$  are the competition coefficients. Eq.(6) is of the Beverton-Holt type for two competition species. When these parameters are  $p$ -periodic one has  $F_{n+p} = F_{n \bmod p}$  and Eq. (6) is said to be  $p$ -periodic. Using the idea of monotone and dissipative maps we prove that, under this scenario, Eq. (6) has a globally asymptotically stable  $p$ -periodic cycle. Moreover, the question of attenuation and resonance will be discussed.

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## On Poncelet-type maps

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The following result is known in projective geometry as the Poncelet's Porism, see [1]: *"Given one ellipse inside another, if there exists one circumscribed  $n$ -gon (simultaneously inscribed in the outer and circumscribed on the inner), then any point on the boundary of the outer ellipse is the vertex of some circumscribed  $n$ -gon."*

The  $n$ -gons in the statement of the above result can be seen as periodic orbits of a map defined from the outer ellipse to itself. Indeed, given one ellipse inside another, we can introduce a Map  $P$  from the exterior one to itself by using the tangent lines to the interior ellipse. This procedure can be extended to any two smooth, nested and convex ovals and we call this type of maps Poncelet's maps. Hence Poncelet's Porism in the dynamical systems setting says: *In the two-ellipses case and when the rotation number of  $P$  is rational there exists a natural number  $n$  such that  $P^n$  is the identity. In other words, the Poncelet's map is conjugated to a rational rotation.*

We study general Poncelet's maps and give several examples of algebraic ovals where the corresponding Poncelet's map has a rational rotation number and it is not conjugated to a rotation. All these cases correspond to integrable maps and the behavior of the rotation number function depending on the energy is very different to the two-ellipses case. We also will provide a new proof of Poncelet's result based on dynamical tools.

This is a joint work with Anna Cima and Armengol Gasull, [2].

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# Global linearization of periodic difference equations

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This talk deals with  $m$ -periodic,  $n$ -th order difference equations . We study whether they can be globally linearized. We give an affirmative answer when  $m = n + 1$  and for most of the known examples appearing in the literature. We also prove some nice features of the  $(n + 3)$ - periodic Coxeter difference equations. The results explained in this talk are based on the joint work [1].

## References and Literature for Further Reading

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## From the creation and annihilation operators to the spatial SIS threshold

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In this study, we consider the spatial stochastic SIS (Susceptible-Infected-Susceptible) epidemic model formulated via creation and annihilation operators. This epidemic model has a continuous phase transition from the absorbing state devoid of infected individuals to a nonequilibrium state of infectivity. In the phase transition the dominant eigenvalue of the evolution operator for the SIS model becomes degenerate, that occurs when the gap between the dominant and the subdominant eigenvalues vanishes. To study the gap value, series expansions in terms of the creation rate have been used. eigenvalues of the evolution operator. The first terms of the series expansion of the gap are computed explicitly with difference equations for the calculation of states.

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# Stabilization of non-linear discrete-time control systems

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In this talk we analyse the stabilization of perturbed non-autonomous discrete-time control systems. Some explicit sufficient conditions are presented under which a class of nonlinear control systems are feedback exponentially stabilizable. We emphasize that, contrary to the usual, we do not use Lyapunov functionals to obtain our results, which can therefore be applied to a general infinite-dimensional setting.

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## Stability analysis of an implicitly defined labor market model

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Until very recently, the pervasive existence of models exhibiting well-defined backward dynamics but ill-defined forward dynamics in economics and finance has apparently posed no serious obstacles to the analysis of their dynamics and stability, despite the problems that may arise from possible erroneous conclusions regarding theoretical considerations and policy prescriptions from such models. A large number of papers have dealt with this problem in the past by assuming the existence of symmetry between forward and backward dynamics, even in the case when the map can not be invertible either forward or backwards. However, this procedure has been seriously questioned over the last few years in a series of papers dealing with implicit difference equations and inverse limit spaces. This paper explores the search and matching labor market model developed by Bhattacharya and Bunzel [1],[2], with the following objectives in mind: (i) to show that chaotic dynamics may still be present in the model for acceptable parameter values, (ii) to clarify some open questions related with the admissible dynamics in the forward looking setting, by providing a rigorous proof of the existence of cyclic and chaotic dynamics through the application of tools from symbolic dynamics and inverse limit theory.

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## Are there simple adaptive heuristics that secure Nash Equilibria?

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Peyton Young (2008) have recently argued that if an adaptive or simple learning rule follows three conditions — (i) it is uncoupled, (ii) each player's choice of action depends solely on the frequency distribution of past play, and (iii) each player's choice of action, conditional on the state, is deterministic — there is no such rule that leads the players' behavior to converge to Nash equilibrium. This result has been widely accepted in the field of game theory, but has not been checked whether it also applies to very complex dynamics that may arise from such games. In this paper we show that there are in fact simple adaptive rules (dependent on past play) that secure convergence, actually fast convergence, in a fully deterministic and uncoupled dynamic game that generates chaotic dynamics. We provide three counterexamples which may raise some doubts about the generality of such result.

## An strongly $F$ -invariant pinched core strip that does not contain any arc of curve.

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The aim of this talk is construct an example of a skew product  $F$  on the cylinder which will have a 2-periodic orbit of curves and an strongly  $F$ -invariant pinched core strip that does not contain any arc of curve.

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## Thresholds for regulatory T cells

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The primary function of the immune system is the protection of the host from pathogen invasion. During such an invasion, T cells under exposure to their specific antigen are activated leading to secretion of growth cytokines (predominantly interleukin 2, denoted IL-2), and expression of the IL-2 receptor which triggers cytokine driven proliferation. Under most circumstances, the immune system is able to successfully remove the pathogen. However, the immune system may also target self antigens (autoimmunity) and cause tissue damage and death. Regulatory T cells (Tregs) limit such autoimmune responses by growth inhibition of T cells. Therefore, the immune system has to achieve a delicate balance between appropriate immune activation and immune response suppression. How such a balance is established and controlled is studied in this presentation. Our motivation is the observation that T cell proliferation through cytokines already has such a control structure; cytokine driven growth exhibits a quorum population size threshold. We propose that Tregs locally adjust these thresholds by inhibiting IL2 secretion. The immune response-suppression axis is then a balance between the local numbers of activated T cells and activated Tregs. We study the effects in the quorum T cell population thresholds by the parameters of the model and we describe the equilibria manifold in a neighborhood of the default values for the parameters and variables.

## Avoidance Control on Time Scales

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We consider dynamic systems on time scales under the control of two agents. One of the agents desires to keep the state of the system out of a given set regardless of the other agent's actions. Leitmann's avoidance conditions are proved to be valid for dynamic systems evolving on an arbitrary time scale.

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## **An asymptotic universal focal decomposition for non-isochronous potentials and some consequences**

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This is joint work with C. A. A. de Carvalho, M. M. Peixoto and A. A. Pinto.

Galileo, in the XVII century, observed that the small oscillations of a pendulum seem to have constant period. In fact, the Taylor expansion of the period map of the pendulum is constant up to second order in the initial angular velocity around the stable equilibrium. It is well known that, for small oscillations of the pendulum and small intervals of time, the dynamics of the pendulum can be approximated by the dynamics of the harmonic oscillator. We study the dynamics of a family of mechanical systems that includes the pendulum at small neighbourhoods of the equilibrium but after long intervals of time so that the second order term of the period map can no longer be neglected. We characterize such dynamical behaviour through a renormalization scheme acting on the dynamics of this family of mechanical systems. The main theorem states that the asymptotic limit of this renormalization scheme is universal: it is the same for all the elements in the considered class of mechanical systems. As a consequence we obtain an universal asymptotic focal decomposition for this family of mechanical systems. Furthermore, we obtain that the asymptotic trajectories have a Hamiltonian character and compute the action of each element in this family of trajectories. We conclude with a description of the utility that the asymptotic universal focal decomposition may have in the computation of propagators in semiclassical physics.

## Dynamic Replicator Equation as a Gradient System

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The bimatrix replicator equation

$$\begin{aligned}x_i^\Delta &= x_i((\mathbf{A}\mathbf{y}^\sigma)_i - \mathbf{x}^T \mathbf{A}\mathbf{y}^\sigma), \quad i = 1, 2, \dots, n, \\y_j^\Delta &= y_j((\mathbf{B}\mathbf{x}^\sigma)_j - \mathbf{y}^T \mathbf{B}\mathbf{x}^\sigma), \quad j = 1, 2, \dots, m\end{aligned}$$

describes an evolutionary dynamics for asymmetric conflicts, cf. [3]. The phase variables  $\mathbf{x}$  and  $\mathbf{y}$  are defined on the interiors of the probability simplexes  $S_n^\circ$  and  $S_m^\circ$ , respectively. In the continuous case and under some assumptions, the equation corresponds to certain gradient system [1, 2].

The contribution introduces a “delta outer derivative”  $\Delta$  of a function. The main result presents necessary and sufficient conditions for existence of a function  $V$  defined on the tangent space to  $S_n^\circ \times S_m^\circ$  such that

$$\langle (\mathbf{x}^\Delta, \mathbf{y}^\Delta), (\boldsymbol{\xi}, \boldsymbol{\eta}) \rangle = \Delta V(\boldsymbol{\xi}, \boldsymbol{\eta}),$$

where  $\langle \cdot, \cdot \rangle$  denotes an inner product in the tangent space.

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## Noninvertible difference equations in the neighbourhood of invariant manifold

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In Banach space  $\mathbf{X} \times \mathbf{E}$  the system of difference equations

$$\begin{aligned}x(t+1) &= g(x(t)) + G(x(t), p(t)), \\p(t+1) &= A(x(t))p(t) + \Phi(x(t), p(t))\end{aligned}\tag{7}$$

is considered. Sufficient conditions under which there is an local Lipschitzian invariant manifold  $u: \mathbf{X} \rightarrow \mathbf{E}$  are obtained. Using this result we prove the asymptotic phase type property and find sufficient conditions of partial decoupling and simplifying of the system of noninvertible difference equations (7).

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## Numerical estimation of the Hölder exponent

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In a certain class of (semi)-discrete dynamical systems  $\mathfrak{F}(\theta_n, x_n)$ , invariant sets with a strange geometry appear. As it is known, the chaoticity of attractors is not necessarily related to its strangeness. For example the iteration of two-dimensional quasi-periodically forced skew product, under certain conditions, gives us Strange Non-Chaotic Attractors. But, *how much strange are these attractors?*

An analytical tool to answer the question is the Hölder exponent to measure the regularity of the object. The aim of this talk is to give a numerical estimate of the Hölder exponent of a concrete example of a Strange Non-Chaotic Attractor using wavelet techniques.

We will briefly introduce the necessary tools for wavelet basis and Besov Spaces. This is a joint work with Prof. Lluís Alsedà.

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## Symbolic dynamics and renormalization of nonautonomous periodic dynamical systems

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The purpose of this paper is to introduce the symbolic formalism, based on kneading theory, that allows us to study the renormalization of nonautonomous periodic dynamical systems.

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## Symplectic structure of Jacobi systems for time scales control problems

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In this talk we study the structure of the Jacobi system for optimal control problems on time scales. Under natural and minimal invertibility assumptions on the coefficients we prove that the Jacobi system is a time scale symplectic system and not necessarily a Hamiltonian system. These new invertibility conditions are weaker than those considered in the current literature. This shows that the theory of time scale symplectic systems, rather than the theory of linear Hamiltonian systems, is fundamental for optimal control problems. Our results new even for the Jacobi equations arising in the time scale calculus of variation and, in particular, for the discrete time calculus of variations and optimal control problems. This is a joint work with Vera Zeidan from Michigan State University.

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## Dynamical systems generated by piecewise continuous maps

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According to a well-known result the collection of all  $\omega$ -limit sets of a continuous map of the interval equipped with the Hausdorff metric is a compact metric space. In the talk, among others, a similar result is given for piecewise continuous maps with finitely many points of discontinuity, if the points of discontinuity are not periodic with respect to some associated maps. We also provide some related results and open problems. The talk is based on a joint work with Franz Hofbauer and Peter Raith.

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## On a triangular maps of type $2^\infty$ with positive topological entropy

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We construct a class of triangular maps of the unique square of type  $2^\infty$ , i.e., such that the periods of periodic points are the powers of 2, which has a minimal set supporting positive topological entropy. The presented results were recently obtained by F. Balibrea, J. Smítal and M. Štefánková.

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## Behaviour of some classes of difference equations

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Behavior of solutions of some special cases to the max-type difference equation

$$x_n = \max \{A_0, A_1 x_{n-p_1}^{r_1} / x_{n-q_1}^{s_1}, A_2 x_{n-p_2}^{r_2} / x_{n-q_2}^{s_2}, \dots, A_k x_{n-p_k}^{r_k} / x_{n-q_k}^{s_k}\}, \quad n \in \mathbb{N}_0,$$

where  $p_i, q_i, i = 1, \dots, k$ , are natural numbers such that  $p_1 < p_2 < \dots < p_k$ ,  $q_1 < q_2 < \dots < q_k$ ,  $k \in \mathbb{N}$ ,  $r_i, s_i \in [0, \infty)$ ,  $i = 1, \dots, k$  and  $A_i \in [0, \infty)$ ,  $i = 0, 1, \dots, k$  are presented in this talk. For example, we show that every positive solution to

$$x_n = \max \{A_1 / x_{n-p_1}^{\alpha_1}, A_2 / x_{n-p_2}^{\alpha_2}, \dots, A_k / x_{n-p_k}^{\alpha_k}\}, \quad n \in \mathbb{N}_0,$$

where  $A_i > 0$ ,  $\alpha_i \in (-1, 1)$ ,  $i = 1, \dots, k$ , converges to  $\max_{1 \leq i \leq k} \{A_i^{1/(\alpha_i+1)}\}$ . We also describe the behavior of well-defined solutions of Eq.  $x_{n+1} = \max\{A_n/x_n, x_{n-1}\}$ ,  $n \in \mathbb{N}_0$ , where  $(A_n)_{n \in \mathbb{N}_0}$  is a two-periodic sequence. Some results concerning the difference equation  $x_{n+1} = \max\{A/x_n, x_{n-k}\}$ ,  $n \in \mathbb{N}_0$ ,  $k \in \mathbb{N}$  are also presented.

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## Change of stability without bifurcation. An Example

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In this talk we analyze in detail an explicit example of a reversible system in which we can find a degenerate subharmonic bifurcation scenario (the so-called *banana scenario*) for which the stability behaviour of the solutions is different from that in a generic banana scenario, and also in contrast with the popular believe (among dynamical systems people) that a change of stability should result in some bifurcation. Namely, we find bifurcating branches of subharmonics along which there is a change of stability (from elliptic to hyperbolic or vice versa), but no further bifurcation. We explain this non-generic behaviour as a consequence of the existence of a certain type of first integral. We also give some numerical evidence supporting the theory.

This talk is based on joint work with Javier Muñoz Almaraz, Emilio Freire and Jorge Galán.

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# Regular Variation on Time Scales with Applications to Dynamic Equations

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We discuss a regular variation on time scales, which extends well studied classic theory of continuous and discrete cases. We show how the recently introduced concept of regular variation on time scales is related to a Karamata type definition. As an application, we consider the second order half-linear dynamic equation

$$(\Phi(y^\Delta))^\Delta - p(t)\Phi(y^\sigma) = 0$$

on time scales and give necessary and sufficient conditions under which certain solutions of this equation are regularly varying. The talk is finished by a few interesting open problems and concluding comments of this themes.

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# Friedrichs extension of operators defined by linear Hamiltonian systems on infinite interval

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We consider a continuous time linear operator  $\mathcal{L}_H(z)(t)$  associated with a linear Hamiltonian system, i.e, with the system

$$x'(t) = A(t)x(t) + B(t)u(t), \quad u'(t) = C(t)x(t) - A^T(t)u(t), \quad t \in [0, \infty), \quad (\mathcal{H})$$

where the real  $n \times n$  matrices  $A(t)$ ,  $B(t)$ , and  $C(t)$  are piecewise continuous on  $[0, \infty)$ , and where  $B(t)$  and  $C(t)$  are symmetric.

We characterize the Friedrichs extension of the minimal operator defined by  $\mathcal{L}_H(z)(t)$  in terms of the recessive (i.e., principal) system of solutions of system  $(\mathcal{H})$ . This generalizes a similar result recently obtained by Zettl in [1] for a class of higher order singular ordinary differential operators and more recently by Marletta and Zettl in [2] for singular differential operators of order  $2n$ . This paper is based on a joint work with Roman Šimon Hilscher in [3].

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International Conference on Difference Equations

# SCIENTIFIC PROGRAM

## SCIENTIFIC PROGRAM

Monday, October 19, 2009

<b>Speakers</b>	<b>Main Talks</b>
<b>KOLYADA</b>	<i>Topology of minimal sets</i>
<b>MISIUREWICZ</b>	<i>Rectangular braids</i>
<b>STRIEN</b>	<i>Iterations of smooth interval maps, 30 years beyond</i>

<b>Speakers</b>	<b>Contributed Talks</b>
<b>COSTA R.</b>	<i>Coexistence of uncountably many attracting sets</i>
<b>DOŠLÝ</b>	<i>Oscillation Theorems for Symplectic Difference Systems</i>
<b>DVOŘÁKOVÁ</b>	<i>Stability of chain recurrent points of a continuous map of the interval</i>
<b>ELYSEEVA</b>	<i>Comparison theorems for Symplectic difference system</i>
<b>FERREIRA H.</b>	<i>1. Bayesian-Nash Equilibria in a Behavior Model</i>
Subs: Gonçalves	<i>2. Universal fluctuations in stock exchange markets</i>
<b>FIŠNAROVÁ</b>	<i>Linearized Riccati technique in discrete half-linear oscillation theory</i>
<b>GOMES</b>	<i>The Pace of Information Diffusion in a Monetary Policy Model</i>
<b>HILGER</b>	<i>Quivers and Difference Equations</i>
<b>MENDES V.</b>	<i>Are there simple adaptive heuristics that secure Nash Equilibria?</i>
<b>PINHEIRO</b>	<i>An asymptotic universal focal decomposition for non-isochronous potentials and some consequences</i>
<b>POSPÍŠIL</b>	<i>Dynamic Replicator Equation as a Gradient System</i>
<b>STEVIC</b>	<i>Behaviour of some classes of difference equations</i>
<b>ZEMÁNEK</b>	<i>Friedrichs extension of operators defined by linear Hamiltonian system on infinite interval.</i>

<b>Timetable - Monday, October 19, 2009</b>	
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8:30	Opening Session – Chair: J.M. Ferreira
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	Chair Alsedà
	Room 1
9:00-9:50	<b>Misiurewicz</b>

10:00-10:20	Coffee Break
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	Chair Yorke
	Room 1
10:20-11:10	<b>Strien</b>
11:20-12:10	<b>Kolyada</b>

12:20-14:00	Lunch
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	Chair Vanderbawhede	Chair Strien
	Room 1	Room 2
14:00-14:25	<b>Stević</b>	<b>Ferreira H.</b>
14:25-14:50	<b>Hilger</b>	<b>Ferreira H.</b>
14:50-15:15	<b>Došlý</b>	<b>Mendes V.</b>
15:15-15:40	<b>Elyseeva</b>	<b>Gomes</b>

15:40-16:00	Coffee Break
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	Chair Stević	Chair Elyseeva
	Room 1	Room 2
16:00-16:25	<b>Fišnarová</b>	<b>Pinheiro</b>
16:25-16:50	<b>Dvořáková</b>	<b>Pospíšil</b>
16:50-17:15	<b>Costa R.</b>	<b>Zemánek</b>

## SCIENTIFIC PROGRAM

Tuesday, October 20, 2009

<b>Speakers</b>	<b>Main Talks</b>
<b>BARREIRA</b>	<i>Quantitative Poincaré Recurrence</i>
<b>LADAS</b>	<i>Open Problems and Conjectures in Difference Equations</i>

<b>Speakers</b>	<b>Contributed Talks</b>
<b>DIBLÍK</b>	<i>Wazewski's method for systems of dynamic equations on time scales</i>
<b>FERNANDES</b>	<i>Synchronization, coupling and mixing time in Markov maps</i>
<b>FERREIRA R.</b>	<i>Calculus of Variations with Discrete Fractional Derivatives</i>
<b>FERREIRO</b>	<i>Strongly inverse positive operators for <math>n</math>th order difference equations</i>
<b>GASULL</b>	<i>On the dynamics of some non-autonomous Lyness type recurrences</i>
<b>GIREJKO</b>	<i>Calculus of variations on time scales with delta, nabla and directional derivatives</i>
<b>HAMAYA</b>	<i>Global stability properties of virus dynamics discrete models</i>
<b>IWASA</b>	<i>Multiple equilibria in a dynamic North-South model</i>
<b>JANG</b>	<i>Discrete host-parasitoid models with Allee effects and age structure in the host</i>
<b>LAITCHOVÁ</b>	<i>Difference and Sum Operators in the Space of Strictly Monotonic Functions</i>
<b>LAUREANO</b>	<i>Efficient synchronization of one-dimensional chaotic maps by a non-symmetric coupling</i>
<b>MEDINA</b>	<i>Stabilization of Non-Linear Discrete-Time Control Systems</i>
<b>PAWŁUSZEWICZ</b>	<i>Avoidance Control on Time Scales</i>
<b>ŠIMON HILSCHER</b>	<i>Symplectic structure of Jacobi systems for time scales control problems</i>
<b>ŠTEFÁNKOVÁ</b>	<i>On a triangular map of type <math>2^\infty</math> with positive topological entropy</i>
<b>VÍTOVEC</b>	<i>Regular Variation on Time Scales with Applications to Dynamic Equations</i>

**Timetable - Tuesday, October 20, 2009**

	Chair Schreiber
	Room 1
9:00-9:50	<b>Ladas</b>

10:00-10:20	Coffee Break
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	Chair Misiurewicz	Chair S. Hilscher
	Room 1	Room 2
10:20-11:10	<b>Barreira</b>	
11:20-11:45	<b>Štefánková</b>	<b>Laitochová</b>
11:45-12:10	<b>Gasull</b>	<b>Ferreiro</b>

12:10-14:00	Lunch
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	Chair Nishimura	Chair Došlý
	Room 1	Room 2
14:00-14:25	<b>Medina</b>	<b>Vítovec</b>
14:25-14:50	<b>Hamaya</b>	<b>Diblík</b>
14:50-15:15	<b>Jang</b>	<b>Ferreira R.</b>
15:15-15:40	<b>Iwasa</b>	<b>Girejko</b>

15:40-16:00	Coffee Break
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	Chair Štefánková	Chair Hilger
	Room 1	Room 2
16:00-16:25	<b>Fernandes</b>	<b>Pawłuszewicz</b>
16:25-16:50	<b>Laureano</b>	<b>S. Hilscher</b>
16:50-17:50	<b>General Meeting ISDE</b>	

## SCIENTIFIC PROGRAM

Wednesday, October 21, 2009

<b>Speakers</b>	<b>Main Talks</b>
<b>FIEDLER</b>	<i>Noninvasive control by delayed feedback</i>
<b>YORKE</b>	<i>Infinitely Many Cascades Must Exist as Chaos Arises in Dimension 2</i>

<b>Speakers</b>	<b>Contributed Talks</b>
<b>BOBOK</b>	<i>Difference equations and entropy of interval maps</i>

### Timetable - Wednesday, October 21, 2009

	Chair Ladas
	Room 1
9:00-9:50	<b>Yorke</b>

10:00-10:20	Coffee Break
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	Chair Barreira
	Room 1
10:20-11:10	<b>Fiedler</b>
11:20-11:45	<b>Bobok</b>

11:45-13:00	Lunch
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13:00	Departure of bus to excursion
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**SCIENTIFIC PROGRAM**  
(No talks this afternoon)  
Wednesday, October 21, 2009

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## SCIENTIFIC PROGRAM

Thursday, October 22, 2009

<b>Speakers</b>	<b>Main Talks</b>
<b>DOWNAROWICZ</b>	<i>Classification of types of convergence of monotone nets of real functions</i>
<b>SCHREIBER</b>	<i>Dispersal mediated persistence in heterogenous environments</i>

<b>Speakers</b>	<b>Contributed Talks</b>
<b>ABDERRAMAN</b>	<i>A linear non autonomous area-preserving map associated to the Szegő recurrence</i>
<b>ALMEIDA</b>	<i>Tilings and Anosov Diffeomorphisms</i>
<b>AL-SHARAWI</b>	<i>A refinement of Sharkovsky's theorem for periodic difference equations</i>
<b>BAPTISTA D.</b>	<i>The structure of kneading curves for Lozi maps</i>
<b>BAŠTINEC</b>	<i>Positive and Oscillating Solutions of Linear Discrete Equations</i>
<b>CORREIA</b>	<i>Markov subshifts and infinite dimension dynamical systems arising from iterated maps</i>
<b>DING</b>	<i>On the dynamics of Lorenz maps</i>
<b>JANKOVSKI</b>	<i>First-order difference equations with nonlinear boundary conditions</i>
<b>KIPNIS</b>	<i>Stability cone for difference equation <math>x(n)=Ax(n-1)+Bx(n-k)</math> with the commuting matrices <math>A,B</math></i>
<b>KOČAN</b>	<i>Entropy, horseshoes and homoclinic trajectories on trees, graphs and dendrites</i>
<b>KUNDRÁT</b>	<i>Remark on numerical nightmare phenomenon</i>
<b>MAÑOSA</b>	<i>On Poincaré-type maps</i>
<b>OLIVEIRA B.</b>	<i>Thresholds for regulatory T cells</i>
<b>REINFELDS.</b>	<i>Noninvertible difference equations in the neighbourhood of invariant manifold</i>
<b>ROMERO S</b>	<i>Numerical estimation of the Hölder exponent</i>
<b>SIMÕES</b>	<i>Symbolic dynamics and renormalization of nonautonomous periodic dynamical systems</i>
<b>SMÍTAL</b>	<i>Dynamical systems generated by piecewise continuous maps</i>

<b>Timetable - Thursday, October 22, 2009</b>
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	Chair Smítal
	Room 1
9:00-9:50	<b>Downarowicz</b>

10:00-10:20	Coffee Break
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	Chair Bobok	Chair Diblík
	Room 1	Room 2
10:20-11:10	<b>Schreiber</b>	
11:20-11:45	<b>Reinfelds</b>	<b>Almeida</b>
11:45-12:10	<b>Kipnis</b>	<b>Mañosa</b>

12:10-14:00	Lunch
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	Chair Kipnis	Chair Reinfelds
	Room 1	Room 2
14:00-14:25	<b>Smítal</b>	
14:25-14:50	<b>Kočan</b>	<b>Al-Sharawi</b>
14:50-15:15	<b>Ding</b>	<b>Jankowski</b>
15:15-15:40	<b>Simões</b>	<b>Kundrát</b>

15:40-16:00	Coffee Break
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	Chair Hüls	Chair Medina
	Room 1	Room 2
16:00-16:25	<b>Baptista D.</b>	<b>Romero S.</b>
16:25-16:50	<b>Correia</b>	<b>Abderraman</b>
16:50-17:15	<b>Oliveira B.</b>	<b>Baštinec</b>

## SCIENTIFIC PROGRAM

Friday, October 23, 2009

<b>Speakers</b>	<b>Main Talks</b>
<b>ELAYDI</b>	<i>Nonautonomous competition models</i>
<b>HÜLS</b>	<i>On r-periodic orbits of k-periodic maps*</i>

\* Special talk – ISDE PRIZE

<b>Speakers</b>	<b>Contributed Talks</b>
<b>BAPTISTA A.</b>	<i>Quadratic dynamics in matrix algebras</i>
<b>CIMA</b>	<i>Real Dynamics of Integrable Birational Maps</i>
<b>FERREIRA M.</b>	<i>Patents in new technologies</i>
<b>HASIL</b>	<i>Critical Higher Order Sturm-Liouville Difference Operators</i>
<b>LUÍS</b>	<i>Nonautonomous periodic Leslie-Gower competition model</i>
<b>MAÑOSAS</b>	<i>Global linearization of periodic difference equations</i>
<b>MARTINS</b>	<i>From the creation and annihilation operators to the spatial SIS threshold</i>
<b>MENDES D.</b>	<i>Stability Analysis of an Implicitly Defined Labor Market Model</i>
<b>MORALES</b>	<i>A strongly F-invariant pinched core strip that does not contain any arc of curve.</i>
<b>VANDERBAUWHEDE</b>	<i>Change of Stability without Bifurcation, An Example</i>

<b>Timetable - Friday, October 23 2009</b>
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	Chair Downarowicz
	Room 1
9:00-9:50	<b>Elydi</b>

10:00-10:20	Coffee Break
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	Chair Kolyada
	Room 1
10:20-11:10	<b>Hüls</b>
11:20-11:45	<b>Vanderbauwhede</b>
11:45-12:10	<b>Cima</b>

12:00-14:00	Lunch
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	Chair Oliveira	Chair Silva
	Room 1	Room 2
14:00-14:25	<b>Luís</b>	<b>Hasil</b>
14:25-14:50	<b>Morales</b>	<b>Martins</b>
14:50-15:15	<b>Manõsas</b>	<b>Baptista A.</b>
15:15-15:40	<b>Ferreira M.</b>	<b>Mendes D.</b>